Building Concepts: Building Expressions

TEACHER NOTES

Lesson Overview

Algebraic Focus: What does it mean to say two expressions are equivalent?

In this lesson students view expressions as objects in their own right and interpret the form and structure of an algebraic expression. Students focus on comparing expressions and establish a foundation for the definition of equivalent expressions. Students also consider how to verify that expressions are equivalent by using the associative and commutative properties of addition and multiplication as well as the distributive property.

Students can use a combination of the associative and commutative properties for addition and multiplication.

Learning Goals

1. Create expressions that satisfy given constraints;
2. identify equivalent expressions and explain why they are equivalent using properties of multiplication and addition;
3. use a variety of strategies, including rewriting an expression and properties of operations, to develop equivalent expressions;
4. understand that a counterexample can disprove a claim, but examples cannot establish a general claim.

Prerequisite Knowledge

Building Expressions is the third lesson in a series of lessons that explore the concept of expressions. This lesson builds on the concepts of the previous lessons. Prior to working on this lesson students should have completed *What is an Exponent?* and *What is a Variable?* Students should understand:

- the order or operations;
- the associative and commutative properties of addition and multiplication;
- the distributive property.

Vocabulary

- **expression:** a phrase that represents a mathematical or real-world situation
- **variable:** a letter that represents a number in an expression
- **constant:** an expression that does not contain a variable
- **associative property:** a law that states that terms can be added or multiplied regardless of how they are grouped
- **commutative property:** a law that states that order does not matter when adding or multiplying numbers
- **distributive property:** allows you to multiply a sum by multiplying each addend separately
- **equivalent expressions:** having the same value for every possible replacement for the variable or variables

Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Building Expressions_Student.pdf
- Building Expressions_Student.doc
- Building Expressions.tns
- Building Expressions_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.

Deeper Dive: These questions are provided for additional student practice, and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

A mathematical expression is a phrase about a mathematical or real-world scenario. Mathematical expressions can be seen and treated as objects in their own right. That process allows students to interpret the form and structure of an algebraic expression as it represents a given context.

Students should become familiar with the structure of an expression, recognizing $2xy + 8$ as the sum of a product and a constant value. In grade 6, they leave behind the symbol "$\times$" for multiplication and come to understand that putting a number and a variable together such as 3*a* implies multiplication between 3 and *a*. Their early use of properties to calculate with numbers, for example seeing $5(47)$ as $5(40+7) = 200+35 = 235$, should help them transition to seeing $4(z+10)$ as $4z+40$ as well as seeing $4z+40$ as $4(z+10)$.

Prerequisite knowledge for working with expressions includes understanding the order of operations and the properties of those operations. Students should have extensive experience in K–5 working with the properties of operations, and in grade 6 they build on this to manipulate algebraic expressions and produce different but equivalent expressions. They should be familiar with the associative and commutative properties of addition and multiplication. They can use an "any order, any grouping" property of addition as a combination of the associative and commutative properties that says a sequence of additions may be calculated in any order and terms may be grouped together any way. A similar statement is true for a sequence of multiplications. Understanding the distributive law is fundamental as it is the only property that connects the operations of multiplication and addition. Students should recognize that

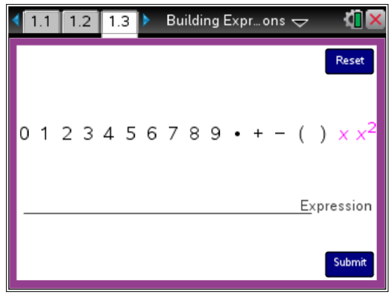
$5a+3b+2a+4b = (a+a+a+a+a)+(b+b+b)+(a+a)+(b+b+b+b)$ and using the "any order any grouping" property will be the same as $7a+7b$, an alternative to $(5+2)a+(3+4)b$.

Note: this counting interpretation works only for whole numbers. Students should eventually see collecting like terms, such as $5a+3b+2a+4b$, as an application of the distributive law.

Note: the questions for pages 1.3 and 1.4 are designed for students without experience with negative integers. Operations with signed numbers are in Grade 7, so in this lesson the main focus is on expressions that do not involve negative rational numbers. If students have had experience with negative integers, the questions can be adapted for use with pages 2.2 and 2.3, where the number line goes from -10 to 10.

Part 1, Page 1.3

Focus: Students reason about values produced by an expression as the variable *x* changes and look for relationships between the structure of the expression and its value.



On page 1.3 students enter an expression by using the touchpad to drag and drop values, variables, and operations. Dragging a selection below the line deselects it. Students may also type values, operations, and variables using the keys. Note that CAPS must be off in order to type the variable *x*.

Submit shows a number line with a dynamic value of *x*. Adjust *x* using the arrow keys.

Edit returns to update/edit the expression.

TI-Nspire Technology Tips

The menu button, menu, accesses page options.

To submit, or return to edit an expression, use the enter button.

To reset the page, use the ctrl del buttons.

Class Discussion

The following questions are designed to review order of operations and engage students in reasoning about the values produced by an expresssion as the variable x changes and in looking for patterns and relationships between the structure of the expression and its value.

Have students…	**Look for/Listen for…**
Use page 1.3 to make the expression $3(4+7)$.	
• ***Move the dot on the number line. Why does the value of the expression stay the same wherever the dot is located?***	Answer: The expression does not contain a variable. It is a *constant* expression because it does not change its value.
• ***Find another way to create an expression that will produce the same result for every value of x.***	Answers will vary. Examples might be $3\cdot 11$ or $3\cdot 4+3\cdot 7$.
Create each of the expressions below. Explain to a partner how you would calculate the result. Check your answer with the file. Create an expression that	
• ***involves numbers, adding, subtracting and parentheses.***	Answers will vary. An example might be $7-(5+3)$ or $17-2+4(6-4)$.
• ***involves sums, products and parentheses.***	Answers will vary. An example might be $5(3-1)+4$.
• ***involves the sum of at least one product, parentheses and subtraction.***	Answers will vary. An example might be $12\cdot 3+8(5-2)$.

Class Discussion (continued)

Suppose you had to tell a classmate who missed school about the "order of operations"—how you calculate a string of operations that consist of addition, subtraction, multiplication, division, exponents, and some parentheses. Use the file on page 1.3 to check your thinking until you have a good way of helping your classmate understand what to do. Create examples to help make what you are saying clear.	Answers will vary. Students should recognize that parentheses signal a grouping that should be done first or that the group should be treated as one quantity and that exponents apply to the base for the exponent and so it is important to be clear about the base. When the operations are a choice between adding (or subtracting) and multiplication (or division), the adding (subtraction) comes before the multiplication (division), but if the problem is all adding and subtracting (or multiplying and dividing) you work from left to right.
Create an expression that satisfies the following conditions. Use the file to check your answers.	
• ***The value of the expression is 0 when x is 4.***	Answers will vary. One possible answer is $x-4$.
• ***The product of x and a number will always be even.***	Answers will vary. One example might be 2*x*.
• ***The product of x and a number will always be odd.***	Answers will vary. One possible answer is $2x+1$.
• ***The result will always be 1 less than a multiple of 3.***	Answers will vary. One possible answer is $3x-1$.
Create the expression $5x+8$ and select Submit.	
• ***Write down four or five of the values of $5x+8$ you get as x varies.***	Answers will vary. They might include 8, 13, 18, 23, …
• ***As x changes by 1 unit, how does the value of the expression change?***	Answer: It increases by 5 for each one-unit change in *x*.
• ***Predict what value of x will make the value of the expression equal to 33. Use the file to check your prediction.***	Answer: $x=5$.

Student Activity Questions—Activity 1

1. **Suri has *x* stamps. The number of stamps some of her friends have is described below.**

   Write an expression for each. Write the number of stamps your expression will produce for several different values of *x* and explain why the numbers make sense in the context of the problem.

   a. **Pat has 5 more than twice as many stamps as Suri.**

      Answer: $2x+5$; One possible example: when $x=1$, the expression produces 7. Suri would have 1 stamp, and Pat would have 7 stamps or 5 more than $2(1)$.

   b. **Kay has 4 less than the number of stamps Suri.**

      Answer: $x-4$; One possible example: when $x=10$, the expression produces 6. When Suri has 10 stamps, Kay has $10-4=6$ stamps.

   c. **Greg has twice as many as three more than the number of stamps Suri.**

      Answer: $2(x+3)$; One possible example: when $x=7$, the expression produces 20. When Suri has 7 stamps, Greg has 20 stamps, twice as many as $3+7$ or twice as many as 10.

   d. **How many stamps does Suri have if Greg has 28 stamps?**

      Answer: 11; The value of the expression is 28 when *x* is 11. So when Greg has 28 stamps, Suri has 11 because $2(11+3)=2(14)=28$.

2. **The class is discussing different kinds of expressions created using whole numbers. Do you agree with their statements? Why or why not?**

   a. **Corry said that if you have $x+3x+5x$, all of the *x*'s have to stand for the same number.**

   b. **Trina thought that if you have $x+3x+5x$, each *x* could stand for a different number.**

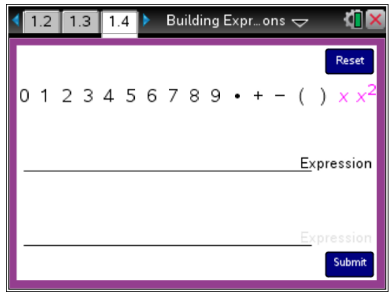
   c. **Sadee said the value of *x* in the expression $x-22$ was 22.**

   d. **Petro claimed that the value of *x* in the expression 2*x* had to be an even number.**

   Answers will vary. Corry is the only one who is correct. Trina is incorrect; if you want to use a different number for the *x* in 3*x* than for the *x* in 5*x* in the same problem, then you have to use a different letter for each of the *x*'s. The value of *x* in $x-22$ can be any number of the given range of values as you can tell from the file. When *x* is 22, $x-22=0$. Petro is mixing the value of the expression with the value of the variable *x*. The value of the expression will always be even because it is a product of 2 times a whole number, *x.*

Part 2, Page 1.4

Focus: Students compare expressions and establish a foundation for the definition of equivalent expressions. Students also consider how to verify that expressions are equivalent by using the associative and commutative properties of addition and multiplication as well as the distributive property.



Page 1.4 functions in the same way as page 1.3.

TI-Nspire Technology Tips

To move between line entries, use the tab tab.

Class Discussion

The following questions focus on comparing expressions and establishing a foundation for the definition of equivalent expressions.

Have students…	Look for/Listen for…
Predict which of the following expressions will have a larger value for the same value of x. Build each expression and Submit to check your prediction.	
• $5x+8$ and $8x+5$	Answer: For values less than 1, $5x+8$ gives greater values than $8x+5$, the two expressions have equal values when $x=1$, and from then on $8x+5$ gives larger values than $5x+8$.
• $4(2x-3)$ and $8x-12$	Answer: The two expressions produce the same values all of the time.
• $10x+10$ and x^2+10	Answer: $10x+10$ is larger until $x=10$, then x^2+10 is larger.

Class Discussion (continued)

Create the given expression on the first blank line and another expression you think will produce the same values on the second blank line. Give a reason for your thinking, and then use the file to check your conjecture. Compare answers with a partner.

- $\boldsymbol{7x+9x}$

  Answers will vary. One possible answer is $16x$ because 7 of something and 9 of that same thing gives you 16 of the things, i.e., $7x = x+x+x+x+x+x+x$ and $9x = x+x+x+x+x+x+x+x+x$, which makes 16 *x*'s. Another answer might be $8x+8x$.

- $\boldsymbol{12x-4x}$

  Answers will vary. One possible answer might be $8x$ because when they are written out you are taking 4 *x*'s away from 12 *x*'s and are left with 8 *x*'s. Another answer might be $13x-5x$.

- $\boldsymbol{5(x+3)}$

  Answers will vary. One possible answer is $5x+15$ because 5 sets of $(x+3)$ is the same as adding 5 *x*'s and 5 threes; i.e., $(x+3)+(x+3)+(x+3)+(x+3)+(x+3) =$ $(x+x+x+x+x)+(3+3+3+3+3)$ Another answer might be $2x+3x+10+5$.

- $\boldsymbol{(x+4)(x+4)}$

  Answers will vary. One possible answer is $(4+x)(4+x)$. Another might be $x(x+4)+4(x+4)$.

In earlier grades, you studied properties associated with the operations of addition and multiplication.

- ***Write down everything you remember about the associative, commutative and distributive properties. Give an example so others will be able to tell what you were thinking. (You can use the file to support your ideas.)***

  Answers will vary. Some might remember that that regrouping was an easy way to do some multiplication problems; others that you can distribute a number to each part of a sum or can distribute a common factor out of two numbers that are to be added.

- ***Share your answer to the question above with a partner. How were your answers alike? Different?***

  Answers will vary depending on student work.

Class Discussion (continued)

- ***Describe how the properties can help justify your answer to*** $5(x+3)$**.**

Answers may vary. Students should mention that they are regrouping and changing the order of the addition.

Student Activity Questions—Activity 2

Teacher Tip: Be sure students notice the key mathematical idea in question 2. It states: *Two expressions are equivalent if they have the same value for every possible replacement for the variable or variables*.

1. Use the files to help answer each of the following:

a. Carey claims that $2x+3x+8$ **will give the same values as** $5x+8$ **as** x **varies. Do you agree with Carey? Why or why not?**

Answer: She is right; the two expressions produce the same values for the same value of x.

b. Tomas says that $1x+4x+8$ **will also give the same values as** $5x+8$**. What would you say to Tomas?**

Answer: He is right as well; the two expressions produce the same values.

c. Find another expression that will produce the same values as $2x+8$ **as** x **changes.**

Answers will vary. Students might include $2x+5+3$ or $7x-5x+8$.

2. Rewriting an expression using properties of addition and multiplication gives a new expression that is equivalent to the original one. Two expressions are *equivalent* if they have the same value for every possible replacement for the variable or variables. Use the file to decide whether the following are statements are equivalent. Find a property to support your thinking.

a. $3x+2$ **and** $2+3x$

b. $2x+3$ **and** $3x+2$

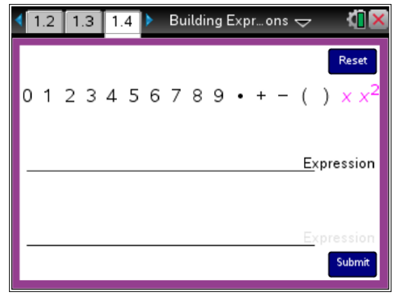
c. $3x+6$ **and** $3(x+2)$

d. $3+2(x+2)$ **and** $2+3(x+2)$

Answer: a and c have equivalent expressions. The two expressions in a illustrate an application of the commutative property, and the two expressions in c illustrate an application of the distributive property. Answer choices b and d are misapplications of the commutative property.

Part 3, Page 1.4

Focus: Students justify their thinking using properties of operations, definitions, and other reasoning strategies.



Class Discussion

These questions provide experience in working with equivalent expressions.

Have students…	Look for/Listen for…
Create the expression $4+2(x+1)$ on the first blank line.	
• ***Do you think the expression $6(x+1)$ will produce the same values as $4+2(x+1)$ as x varies? Explain why or why not.***	Answers will vary. Some may recognize from the work with numbers and their answer to question 3 about the order of operations that you do the multiplication before the addition so $6(x+1)$ should not produce the same value as $4+2(x+1)$. Others might not see this connection yet.
• ***Create $6(x+1)$ on the second blank line and Submit. Move the dot to change the variable. Are $6(x+1)$ and $4+2(x+1)$ equivalent? Why or why not?***	Answer: They are not equivalent because the values of the two expressions are different for the same value of *x*.
• ***Think about what you know about the order of operations. How can this help you decide whether the two expressions are equivalent?***	Answers will vary. The order of operations says to do multiplication before addition so multiplying the parentheses by 2 should have been done before adding the 4 and 2.
Create the two expressions $x \cdot x^2$ and $2x^2$.	
• ***Find the value of each when $x=2$.***	Answer: 8
• ***Would you say the two expressions are equivalent? Why or why not.***	Answer: The expressions are not equivalent because they only give the same value when $x=2$. For other values of *x*, they two expressions give different results.

Class Discussion (continued)

Some statements can be true for many values of a variable or variables but not true for others. For example, "multiplication" makes things larger is true if you are multiplying whole numbers greater than 1. Using definitions and the properties of addition and multiplication can help prove expressions are equivalent that seem to be equivalent because they produce the same values for a set of variables.

- ***Tina claims that $2x+6$ is equivalent to $4+2(x+1)$. Do you agree with her? Why or why not? Use the file to support your thinking.***

  Answer: Yes, the two expressions seem to give the same values for a given value of *x*.

- ***Find a mathematical property to justify your answer to the question above.***

  Answer: The distributive property says that $4+2(x+1)=4+(2x+2)$ and by the "any order, any grouping" property for addition, you can group the 4 and the 2 to get $6+2x$.

Student Activity Questions—Activity 3

1. **Use the file to help you think about whether the two expressions are equivalent. Then try to find a mathematical justification for your answer.**

   a. $5x+4x$ **and** $(5+4)x$

   Answer: These expressions are equivalent by the distributive property; the values of the expressions are the same as the value of x varies.

   b. $8+2x$ **and** $2(4+x)$

   Answer: They values for the two expressions are the same as x varies. The distributive property where 2 is a common factor makes these expressions equivalent.

   c. $4(2x+5)$ **and** $8x+5$

   Answer: The values of the two expressions are different for the same value of x. These are not equivalent because you cannot group half of a multiplication problem about two things with only one of the things. The 5 also has to be multiplied by the 4.

   d. $(3x+5)+(4x-1)$ **and** $7x^2+4$

   Answer: The values of the two expressions are different for the same value of x. These expressions are not equivalent because in the first expression you are not multiplying any x's together, but in the second expression you have x^2 which is $x \cdot x$.

Student Activity Questions—Activity 3 (continued)

2. **Sallee says that $3x+7$ is equivalent to all of the following. Do you agree with her? Use the file and the structure of the expression to support your reasoning.**

   a. $x+x+x+7$

   Answer: This is equivalent because the values of the expression are the same as the values of $3x+7$ for any x; adding 3 x's is the same as 3x.

   b. $x \cdot x \cdot x+7$

   Answer: This is not equivalent because the values of the expression are different from those of $3x+7$ for the same value of x.

   c. $10x$

   Answer: This is not equivalent because the values of the expression are different from those of $3x+7$ for the same value of x.

   d. $x+2x+7$

   Answer: This is equivalent because the values of the expression are the same as those for $3x+7$ for a given value of x; by counting you can tell that adding one x and two x's is the same as 3x's.

   e. $3(x+3)-2$

   Answer: This is equivalent because the values of the expression are the same as the values of $3x+7$ for a given value of x; distributing 3 to the x and to the 3 produces $3x+9-2$. From left to right, $9-2=7$ so the $3(x+3)-2$ is equivalent to $3x+7$.

3. **For each of the following expressions, find two equivalent expressions. (Use the file to check your thinking.) Explain why you think the expressions are equivalent.**

   a. $(x+7)+(4x+2)$

   b. $5x^2-3$

   c. $2x(8x+7)+(3x-5)$

   d. $25+2(x+5)$

   Answers will vary. Possible answers:

   a. $5x+9$; $2x+3x+9$

   b. $2x^2+3x^2-3$; $5x^2-2-1$

   c. $13x+2$; $(10x+1)+(3x+1)$

   d. $35+2x$; $x+x+30+5$

Deeper Dive — Page 1.3

Create each expression and Submit. Describe the value of the expression as x varies.

- $4x+4$ — Answers will vary. One possible description might be multiples of 4.
- $2x+3x+1$ — Answers will vary. A possible response might be 1 more than a multiple of 5.
- $7(x-2)$ — Answers will vary. One possible response is a multiple of 7.
- $x+9x$ — Answers will vary. One possible response might be a multiple of 10.

Deeper Dive — Page 1.4

Build one expression that satisfies both conditions.

- ***The value of the expression is 18 when x is 6 and 6 when x is 2.*** — Answers will vary. Students may recognize that (6, 18) and (2, 6) come from a set of ratios equivalent to the ratio 1:3. One corresponding expression would be 3*x*.
- ***The value of the expression is 10 when x is 5 and 7 when $x=2$.*** — Answers will vary. $x+5$ satisfies the requirements.

Identify any of the following expressions that are equivalent. Give a reason for your answers.

a. $1\cdot x$	**b.** x^2	**c.** $x\cdot x$
d. $x+x$	**e.** x	**f.** $2x$

Answer: *a* and *e* are equivalent because multiplying a number by 1 is the same as the number; *b* and *c* are equivalent by the definition of exponent; *d* and *f* are equivalent because adding two *x*'s is the same as 2 times *x.*

For each of the following find an equivalent expression that does not use parentheses. Be ready to explain why your expression is equivalent to the given expression.

- $3(5x-4)+4x$ — Answers will vary. Possible answer: $15x-12+4x$.
- $x(4x+2)$ — Answers will vary. Possible answer: $4x^2+2x$.
- $(3x^2+14)+(7x^2-8)$ — Answers will vary. Possible answer: $10x^2+6$.
- $5x^2+x(x-4)$ — Answers will vary. Possible answer: $5x^2+x^2-4x$.

Deeper Dive — Page 1.4 (continued)

Use the file to identify each statement as true or false. Decide whether an example can be used to justify your reasoning or whether you need to reason from properties and mathematical definitions.

- $\boldsymbol{(x+3)(x+3)=x^2+9}$

  Answer: False; an example such as $(5+3)(5+3)=(8)(8)=64$, will show that it is not true.

- $\boldsymbol{4x^2+2x^2=3x^2+3x^2}$

  Answer: True, because you can distributive out an x^2 from each expression and get $(4+2)x^2$ and $(3+3)x^2$, so both expressions are equal to $6x^2$.

- $\boldsymbol{7x^2-x^2=7}$

  Answer: False, using an example you can see that $7(9)-9=63-9=54\neq 7$

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Kara claims that the expression x^2+1 results in an even number for all integer values of x. Which value of x shows that Kara's claim is incorrect?

   a. $x=5$ b. $x=3$ c. $x=0$ d. $x=1$

   Texas TAKS grade 9, 2009

   Answer: c) $\boldsymbol{x=0}$

2. Which expression is equivalent to 3 times the sum of x squared and 7?

   a. $3x^2+7$ b. $(3x+7)^2$ c. $3(x+7)^2$ d. $3(x^2+7)$

   Texas TAKS grade 9, 2009

   Answer: d) $\boldsymbol{3(x^2+7)}$

3. Which expression is equivalent to $4(3+x)$?

   a. $12+x$ b. $7+x$ c. $12+4x$ d. $12x$

   TIMSS 2011, Grade 8

   Answer: c) $\boldsymbol{12+4x}$

4. Which of these is equal to $3p^2+2p+2p^2+p$?

   a. $8p$ b. $8p^2$ c. $5p^2+3p$ d. $7p^2+p$

   TIMSS 2011, Grade 8

   Answer: c) $\boldsymbol{5p^2+3p}$

5. Which expression is equivalent to $8(x-2)+4(3-x)-2x$?

   a. $5x+28$ b. $2x-4$ c. $2x+4$ d. $12x+12$

   PAARC Practice Test, 2014

   Answer: b) $\boldsymbol{2x-4}$

6. There were m boys and 23 girls in a parade. Each person carried 2 balloons. Which of these expressions represents the total number of balloons that were carried in the parade?

   a. $2(m+23)$ b. $2+(m+23)$ c. $2m+23$ d. $m+2(23)$

   adapted from TIMSS Grade 8 2011

   Answer: a) $\boldsymbol{2(m+23)}$

Student Activity Solutions

In these activities you will look for relationships between the structure of the expression and its value and verify that expressions are equivalent by using the associative and commutative properties of addition and multiplication, as well as the distributive property. After completing the activities, discuss and/or present your findings to the rest of the class.

Activity 1 [Page 1.3]

1. Suri has *x* stamps. The number of stamps some of her friends have is described below.

   Write an expression for each. Write the number of stamps your expression will produce for several different values of *x* and explain why the numbers make sense in the context of the problem.

   a. Pat has 5 more than twice as many stamps as Suri.

   Answer: $2x+5$; One possible example: when $x=1$, the expression produces 7. Suri would have 1 stamp, and Pat would have 7 stamps or 5 more than $2(1)$.

   b. Kay has 4 less than the number of stamps Suri.

   Answer: $x-4$; One possible example: when $x=10$, the expression produces 6. When Suri has 10 stamps, Kay has $10-4=6$ stamps.

   c. Greg has twice as many as three more than the number of stamps Suri.

   Answer: $2(x+3)$; One possible example: when $x=7$, the expression produces 20. When Suri has 7 stamps, Greg has 20 stamps, twice as many as $3+7$ or twice as many as 10.

   d. How many stamps does Suri have if Greg has 28 stamps?

   Answer: 11; The value of the expression is 28 when x is 11. So when Greg has 28 stamps, Suri has 11 because $2(11+3)=2(14)=28$.

2. The class is discussing different kinds of expressions created using whole numbers. Do you agree with their statements? Why or why not?

   a. Corry said that if you have $x+3x+5x$, all of the *x*'s have to stand for the same number.

   b. Trina thought that if you have $x+3x+5x$, each *x* could stand for a different number.

   c. Sadee said the value of *x* in the expression $x-22$ was 22.

   d. Petro claimed that the value of *x* in the expression 2*x* had to be an even number.

   Answers will vary. Corry is the only one who is correct. Trina is incorrect; if you want to use a different number for the x in 3x than for the x in 5x in the same problem, then you have to use a different letter for each of the x's. The value of x in $x-22$ can be any number of the given range of values as you can tell from the file. When x is 22, $x-22=0$. Petro is mixing the value of the expression with the value of the variable x. The value of the expression will always be even because it is a product of 2 times a whole number, x.

Activity 2 [Page 1.4]

1. Use the files to help answer each of the following:

   a. Carey claims that $2x+3x+8$ will give the same values as $5x+8$ as x varies. Do you agree with Carey? Why or why not?

   Answer: She is right; the two expressions produce the same values for the same value of x.

   b. Tomas says that $1x+4x+8$ will also give the same values as $5x+8$. What would you say to Tomas?

   Answer: He is right as well; the two expressions produce the same values.

   c. Find another expression that will produce the same values as $2x+8$ as x changes.

   Answers will vary. Students might include $2x+5+3$ or $7x-5x+8$.

2. Rewriting an expression using properties of addition and multiplication gives a new expression that is equivalent to the original one. Two expressions are *equivalent* if they have the same value for every possible replacement for the variable or variables. Use the file to decide whether the following are statements are equivalent. Find a property to support your thinking.

   a. $3x+2$ and $2+3x$

   b. $2x+3$ and $3x+2$

   c. $3x+6$ and $3(x+2)$

   d. $3+2(x+2)$ and $2+3(x+2)$

   Answer: a and c have equivalent expressions. The two expressions in a illustrate an application of the commutative property, and the two expressions in c illustrate an application of the distributive property.

Activity 3 [Page 1.4]

1. Use the file to help you think about whether the two expressions are equivalent. Then try to find a mathematical justification for your answer.

   a. $5x+4x$ and $(5+4)x$

   Answer: These expressions are equivalent by the distributive property; the values of the expressions are the same as the value of x varies.

   b. $8+2x$ and $2(4+x)$

   Answer: They values for the two expressions are the same as x varies. The distributive property where 2 is a common factor makes these expressions equivalent.

   c. $4(2x+5)$ and $8x+5$

   Answer: The values of the two expressions are different for the same value of x. These are not equivalent because you cannot group half of a multiplication problem about two things with only one of the things. The 5 also has to be multiplied by the 4.

d. $(3x+5)+(4x-1)$ and $7x^2+4$

Answer: The values of the two expressions are different for the same value of x. These expressions are not equivalent because in the first expression you are not multiplying any x's together, but in the second expression you have x^2 *which is* $x \cdot x$.

2. Sallee says that $3x+7$ is equivalent to all of the following. Do you agree with her? Use the file and the structure of the expression to support your reasoning.

a. $x+x+x+7$

Answer: This is equivalent because the values of the expression are the same as the values of $3x+7$ *for any x; adding 3 x's is the same as 3x.*

b. $x \cdot x \cdot x+7$

Answer: This is not equivalent because the values of the expression are different from those of $3x+7$ *for the same value of x.*

c. $10x$

Answer: This is not equivalent because the values of the expression are different from those of $3x+7$ *for the same value of x.*

d. $x+2x+7$

Answer: This is equivalent because the values of the expression are the same as those for $3x+7$ *for a given value of x; by counting you can tell that adding one x and two x's is the same as 3x's.*

e. $3(x+3)-2$

Answer: This is equivalent because the values of the expression are the same as the values of $3x+7$ *for a given value of x; distributing 3 to the x and to the 3 produces* $3x+9-2$. *From left to right,* $9-2=7$ *so the* $3(x+3)-2$ *is equivalent to* $3x+7$.

3. For each of the following expressions, find two equivalent expressions. (Use the file to check your thinking.) Explain why you think the expressions are equivalent.

a. $(x+7)+(4x+2)$

b. $5x^2-3$

c. $2x(8x+7)+(3x-5)$

d. $25+2(x+5)$

Answers will vary. Possible answers:

a. $5x+9$; $2x+3x+9$;

b. $2x^2+3x^2-3$; $5x^2-2-1$

c. $13x+2$; $(10x+1)+(3x+1)$

d. $35+2x$; $x+x+30+5$