Building Concepts: Visualizing Systems of Linear Equations

TEACHER NOTES

Lesson Overview

In this TI-Nspire lesson, students will reason about solutions to systems of equations while modeling the systems on mobiles.

A sense of the relationship among variables under different constraints can be developed in the context of mobiles.

Learning Goals

1. Write algebraic equations, using variables, that represent the relationship among the values of shapes in a mobile;
2. associate moves that preserve balance in a mobile with algebraic moves that preserve the solutions to a given equation;
3. find solutions to two or more linear equations involving more than one variable.

Prerequisite Knowledge

Visualizing Systems of Linear Equations is the fifteenth lesson in a series of lessons that explores the concepts of expressions and equations. In this lesson, students use mobiles as a context for reasoning about solutions to systems of linear equations, where the solutions are rational numbers. They will experiment with different weights for the shapes that will produce a target for a balanced mobile. Prior to working on this lesson, students should have completed *Equations and Operations*, *Solving Equations*, *Using Structure to Solve Equations,* and *Visualizing Equations Using Mobiles.*

Students should understand:

- the concept of solution-preservation moves and
- how to use the highlight method to solve equations.

Vocabulary

- **equation:** a statement that the value of two mathematical expressions are equal.
- **system of linear equations in two variables:** comprises two or more equations where a common solution to the equations is sought

Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Building Concepts: Visualizing Systems of Linear Equations

TEACHER NOTES

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Visualizing Systems of Linear Equations_Student.pdf
- Visualizing Systems of Linear Equations_Student.doc
- Visualizing Systems of Linear Equations.tns
- Visualizing Systems of Linear Equations_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet also can be completed as a larger group activity, depending on the technology available in the classroom.

Deeper Dive: These questions are provided for additional student practice and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Building Concepts: Visualizing Systems of Linear Equations

TEACHER NOTES

Mathematical Background

In *Visualizing Equations Using Mobiles*, students reasoned about solutions to linear equations in the context of balancing mobiles with the weights of different shapes, where the solutions were whole numbers. In this lesson, students revisit mobiles as a context for reasoning about solutions to systems of linear equations, where the solutions are rational numbers. For mobiles with up to four arms, each containing a number of shapes, students experiment with different weights for the shapes that will produce a target weight or value (which can be any rational number) for a balanced mobile. The relationships among the shapes on the mobiles can be represented algebraically as linear equations; for example, three triangles on one arm must balance a triangle and a circle on the other arm where each of the arms must have the same total weight, i.e., $3T = T + C$ where *T* is the value of a triangle and *C* the value of a circle. As in the earlier lesson, the relationships among the shapes on the mobile can be mapped to equations.

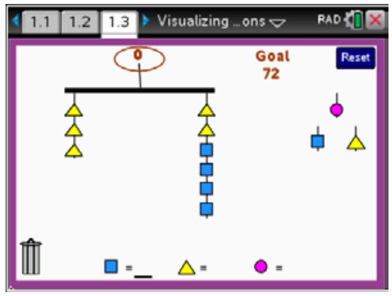
After students become familiar with reasoning about the mobiles and how different configurations can be made to balance using rational numbers, they consider a balanced mobile for which some of the values of the shapes are specified and others are unknown. Students revisit the solution-preserving moves developed in *Equations and Operations* in the context of a balanced mobile, adding or taking away the same shape from both sides to keep the mobile balanced. They also investigate mobiles that can be represented by a system of linear equations involving more than one variable. The systems can be solved by the highlight method from *Using Structure to Solve Equations* or by using solution-preserving moves.

Part 1, Page 1.3

Focus: Associate a mobile with equations and revisit the idea of balancing equations by finding values for the missing shapes that will balance the mobiles.

On page 1.3, tab or select the space by a shape then enter a value, which can be positive, negative, or a fraction. A shape can be moved or added by dragging or by up arrow, tabbing to the selected shape and using the right/left arrows to move the shape to an arm or off the mobile.

Reset returns to the original mobile.



TI-Nspire Technology Tips

menu accesses page options.

tab cycles through the triangle, square and circle.

ctrl del resets the page.

Class Discussion

Have students…	Look for/Listen for…
The objective is to find values for the shapes that will keep the two sides balanced for a given total value for both arms of the mobile. Explain your reasoning for each of the following.	
• ***Find values for the triangle and the square if the total value for the mobile is*** -24***. Use*** **tab** ***or*** **menu> Set Mobile Values** ***to adjust the goal.***	Answer: The left arm of the mobile has to have value -12 if the mobile is balanced, so a triangle will have value -4. The right arm of the mobile will also be -12. The two triangles are worth -8, so four squares are -4 and each square is -1.
• ***Make a conjecture about the values of the shapes if the total value of the mobile is*** -48 ***and explain your thinking. Check your conjecture using the mobile.***	Answer: Doubling the value of each shape will make a triangle -8 and the square -2. This would make the left arm of the mobile have value -24 and the right arm $-16+-8=-24$ for a total value of -48.
• ***If the total value of the mobile is*** -12***, what will the value of each shape be?***	Answer: Reasoning in the same way as above, -12 is half -24, so taking half of the value of the triangle would make it -2 and the half of the square would be $-\frac{1}{2}$. This would make each arm worth -6, for a total value of -12.

Class Discussion (continued)

- ***If* T *equals the value of a triangle and* S *equals the value of a square, write two equations that represent a balanced mobile.***

  Answers may vary: The three triangles could be written as $3T$ and the right arm equals the left arm when the mobile is balanced so $3T = -12$; $2T + 4S = -12$, $3T + 2T + 4S = -24$, or $3T = 2T + 4S$.

Consider the mobile on page 1.3.

- ***If* S *is a negative value, does* T *have to have a positive or negative value? Explain your reasoning.***

  Answer: T has to be negative because one T equals four squares and if a square is negative, T will be negative also.

- ***Sami claimed that as long as the value of four squares was the same as the value of one triangle, both bars would balance. Do you agree with Sami? Why or why not?***

  Answer: Sami is correct because if you take two triangles from either arm you are using a solution-preserving move that will keep the arms balanced, so the remaining triangle must have the same value as four squares.

- ***Use your reasoning in the question above to create two different examples where the mobile is balanced. Give the total value of the mobile in each case.***

  Answers will vary. If $S = -3$, $T = -12$, and the total value of the mobile will be -36. If $S = \frac{1}{2}$, $T = 2$, and the total value of the mobile will be 6.

- ***Thor suggested letting the value of a square,* S*, be -20. Since there are 4 times as many triangles as squares so $4T = S$. Thus, $T = -5$ since $4(-5) = -20$. Do you agree or disagree with Thor? Explain your reasoning.***

  Answer: Thor has the variables mixed up. You need $4S = T$ or 4 times the value of S to get the value of T. So, the value of T would be $4(-20) = -80$, not -5.

Student Activity Questions—Activity 1

1. Add a circle to the left arm of the mobile.

a. Set the total value to 36, and determine values for the circle, triangle, and square where at least one of the values is negative.

Answers may vary: $T = 7$, $C = -3$, and $S = 1$ will make both sides 18 for a total value of 36.

b. Write an equation representing the relationships between the arms of the mobile. Be sure to define the variables in your equations.

Answer: If $T =$ the value of a triangle, $S =$ the value of a square, and $C =$ the value of a circle, the equation would be: $3T + C = 2T + 4S$.

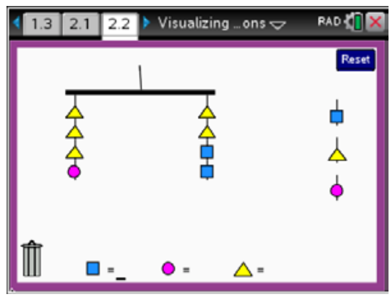
Student Activity Questions—Activity 1 (continued)

c. Is there a unique (only one) value for each of the shapes represented by your equation in b) that makes the statement true? Why or why not? Give an example to support your reasoning.

Answer: As long as $T + C = 4S$, the arms will balance. You can get lots of different values that will make $T + C = 4S$, for example, $S = 2$, $T = 3$, and $C = 5$ or $S = -6$, $T = -20$, and $C = -4$.

Part 2, Page 2.2

Focus: Use solution-preserving moves to find unknown values of the shapes for a mobile when some of the shapes have fixed values.



On page 2.2, enter a value for one or two of the shapes the same way as on page 1.3. Select or tab to **Submit**, which creates a balanced mobile by allocating a hidden value for the unassigned shapes. Move shapes as on page 1.3 to find the unknown value(s) of a shape(s).

Select **Check** to be sure your work is correct. Hovering over a shape in the mobile or tabbing through the shapes displays the value assigned to that shape.

Reset returns to the original mobile. **menu> mobiles** can be used to choose new mobiles.

TI-Nspire Technology Tips

menu accesses page options.

tab cycles through the triangle, square, and circle.

ctrl del resets the page.

Class Discussion

Have students…	Look for/Listen for…
On page 2.2, enter a value for one or more of the shapes and Submit. Then find values for the missing shape(s). Explain your reasoning in each case.	
• ***Let a circle have value -2 and a square value $-\frac{1}{2}$.***	Answer: If T is the value of a triangle, $T = 1$ because by taking two triangles from each arm, a triangle and a circle are the same as two squares ($T + C = 2S$), which means a triangle and a circle together equal -1 ($T + C = -1$). If a circle is worth -2, the triangle has to be 1 ($T - 2 = -1$).
• ***Reset. Add a square on the left arm. A circle has value -5, and the triangle has value 2.***	Answer: $3T + C + S = 2T + 2S$, where S is the value of the square. Take a square and 2 triangles from each arm to get $2 + -5 = S$, which makes $S = -3$.

Class Discussion (continued)

- **Reset.** ***Create the mobile $T + C = S$. A circle has value -5 and triangle value 2. How does this mobile compare to the mobile in the previous question?***

  Answer: A square, S, is -3. $T + C = S$ is the mobile you get if you take two triangles and one square from each arm of the original mobile.

Reset*. Select* Mobile 4*. Let $S = 15$ and $T = -14$.*

- ***Write an equation for the mobile, then find the value of a triangle.***

  Answer: Equation: $T + 2C + S = S + 4C$. Removing two circles and a square from each side gives $2C = T$ or $2C = -14$, so $C = -7$.

- ***How will your answer to the previous question change if the triangle has value -14 and square value 3? Explain your reasoning.***

  Answer: The value of C still will be -7 because the value of S does not matter. A square can have any value because it will be removed from both arms.

- ***How many different values for* C *and* S *can you find that will make mobile 4 balance if $T = -14$? Check your answer using the TNS activity.***

  Answer: There will be an infinite number of values for S because the square can be anything, but C will always be -7.

Student Activity Questions—Activity 2

1. **Use *menu> Mobiles* to select the indicated mobile. Write an equation that represents the relationship among the shapes and find the value of the missing shapes.**

   a. ***Reset*. Select *Mobile 2*. Let circle $= -4$ and square $= 16$. Find the value of a triangle.**

   Answer: Equation: $2T + 2C = 2S + T + C$. triangle $= 36$. Removing a triangle and a circle from each arm gives the mobile $T + C = 2S$. If $S = 16$ and $C = -4$, then $T - 4 = 32$, so $T = 36$.

   b. ***Reset*. Select *Mobile 3*. Let $T = -12$ and $S = 5$.**

   Answer: Equation: $2C + 2S + T = 2T + S$. Taking a square and a triangle from both sides, you get $2C + S = T$ or $2C + 5 = -12$. By the highlight method, $C = -\frac{17}{2}$.

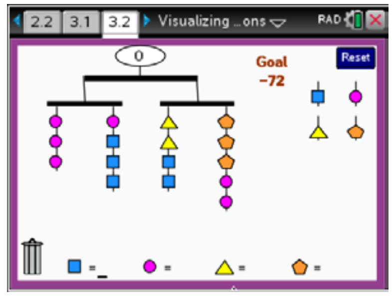
   c. ***Reset*. Select *Mobile 3*. Replace one triangle on the right arm of Mobile 3 with a circle. Let $C = -4$ and $T = 1$.**

   Answer. Equation: $2C + 2S + T = T + C + S$. Taking a triangle, a circle, and a square from both sides gives the mobile $C + S = 0$. If $C = -4$, then $S = 4$.

Part 3, Page 3.2

Focus: Develop a sense of how to use the relationships among the shapes of a mobile to find values that will balance the mobile whether for a given total value or for a mobile with some values fixed.

Page 3.2 functions in the same way as page 1.3.



Class Discussion

Have students…	Look for/Listen for…
Determine values for the shapes on the mobile that will balance the mobile and reach the given total value.	
• ***For the mobile on page 3.2, the total value should be -72.***	Answer: Each arm will have value -18, so 3 circles is -18 and $C=-6$, which makes 3 squares -12, so from $3S=-12$, a square is -4. $-8+2T=-18$, so $T=-5$ and from $-12+3P=-18$. $P=-2$. Thus, $S=-4$, $C=-6$, $T=-5$, and $P=-2$.
• **Reset*. Select* menu> Mobiles> mobile 2*. The total value is -40.***	Answer: P and T have to be opposites, so by choosing values that are opposites and testing values knowing that each arm has value -10, $P=2$, $T=-2$, $C=-4$, and $S=-6$.

Student Activity Questions—Activity 3

1. **Determine values for the shapes on the mobile that will balance the mobile and reach the given total value.**

   a. ***Reset*. Select *menu> Mobiles> mobile 3*. The total value is 48.**

   Answer: The only way a triangle, square, and pentagon can make 12 and that a triangle and 2 squares and a pentagon can make 12 is if a square is 0. Then three squares equals 12 so one square is 4, which makes a pentagon 8. Using those values, $16+3C=12$, so $C=-\frac{4}{3}$. Thus, $S=0$, $T=4$, $P=8$, and $C=-\frac{4}{3}$

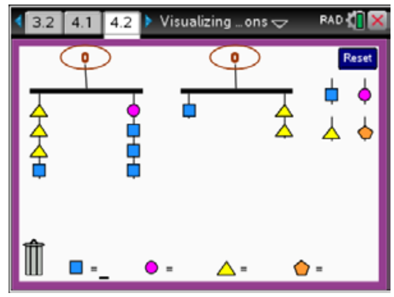
Student Activity Questions—Activity 3 (continued)

b. Consider the mobile in the previous question. Does it make any difference which of the arms have which of the expressions? Why or why not?

Answer: It does not make any difference because each of the arms has to have value 12 in order to have a total value of 48.

Part 3, Page 4.2

Focus: Develop a sense of how to use the relationships among the shapes of a mobile to find values that will balance the mobile whether for a given total value or for a mobile with some values fixed.



Page 4.2 functions in the same way as pages 1.3 and 3.2.

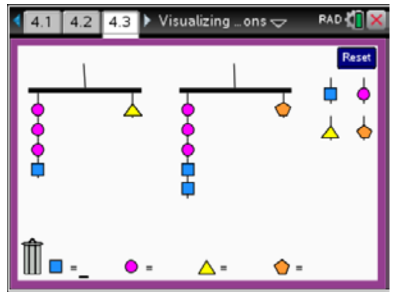
Class Discussion

Have students…	Look for/Listen for…
In each case below, find the value of the square, triangle, and circle.	
• ***Make the mobile on the left have value -40 and the mobile on the right -16.***	Answer: Square has value -8, triangle value -4, and circle value 4.
• ***Add a triangle to the right arm of the right mobile. Make the mobile on the left have value -4 and the mobile on the right have value -2.***	Answer: The square has value -1, triangle has value $-\frac{1}{3}$, and circle 1.
• ***Create a mobile on the left that would represent the equation $2C+T=S$ and a mobile on the right representing the equation $2C+3T=P$ and T represents the value of a triangle, C the value of a circle. If $S=-5$ and $P=-21$ and the value of the mobile on the left is -10 and of the mobile on the right -42, find values for the shapes that make each mobile balance.***	Answer: Two circles and a triangle are on both the left mobile and the right mobile, but the difference between the square on one mobile, -5, and the pentagon on the other, -21, is -16, which means two triangles are -16, $T=-8$ and $C=\frac{3}{2}$.

Part 4, Page 4.3

Focus: Develop a sense of how to use the relationships among the shapes of a mobile to find values that will balance the mobile whether for a given total value or for a mobile with some values fixed.

Page 4.3 functions in the same way as page 2.2, except that values for at least two shapes must be submitted in order to begin.



Teacher Tip: Note that actually moving the shapes in the TNS activity opens up ways of reasoning, often in the form of the cover up or highlight method (finding missing factors or addends) used in earlier activities for solving equations that purely algebraic manipulation does not. Thus, be sure that students actually build the mobiles and explain how they use the mobiles to reason about solutions.

Class Discussion

On page 4.3, you can enter a value for one or more of the shapes and Submit to create a mobile with hidden values for the other shapes. For each of the following explain how you found the values of C and S.

- ***For the system on page 4.3, let $T = -7$ and $P = 1$.***

Answer: $C = -5$ and $S = 8$. Explanations may vary. One strategy is to replace $3C + S$ on the right mobile by the value of T to get $T + S = P$ or $-7 + S = 1$, so $S = 8$. Knowing that $S = 8$, then $3C + 8 = -7$. Asking what number added to 8 makes -7 gives -15, so $3C = -15$ and $C = -5$.

- ***Reset. Leave the left mobile as $3C + S = T$ and change the right mobile to $C + P = S$. Let $T = -2$ and $P = 2$. Find the values of C and S.***

Answer: $C = -1$ and $S = 1$. Replace the square in the left mobile by a circle and a pentagon. This gives four circles and a pentagon making a triangle or $4C + 2 = -2$. Thinking about what added to 2 makes -2 gives -4, so 4C is -4 or $C = -1$, making $S = 1$.

Class Discussion (continued)

- ***Reset and create the system $3C + S = T$ and $T + 3S = P$, where $T = -5$ and $P = 16$.***

Answer: $C = -4$ and $S = 7$. One strategy is to think that in the right mobile 16 is the sum of -5 and what number; the number is 21. So three squares makes 21, so a square is 7. Thinking the same way, -5 is the sum of 7 and what number. The number is -12, so 3 circles make -12 so one circle is -4.

- ***Reset and create the system $3C + S = T$ and $2C = S$, where $T = \frac{5}{2}$. Find the values of C and S.***

Answer: $C = \frac{1}{2}$ and $S = 1$. Replace the square by two circles on the left mobile to get five circles $= \frac{5}{2}$ so one circle would be $\frac{1}{2}$. This makes the square, which is two circles, 1.

For each of the following, create a "mobile" for the problem and use it to find the unknown values. Explain your reasoning in each case.

- ***Two notebooks and four pens cost $32. One notebook and three pens cost $20. Find the cost of one notebook and of one pen.***

Answer: If *N* is the cost of a notebook and *P* is the cost of a pen, then the "mobiles" would be represented by the equations $2N + 4P = 32$ and $N + 3P = 20$. The cost of a pen is $4 and of a notebook $8. Explanations will vary. $N + N + P + P + P + P = 32$ and $N + P + P + P = 20$. Replace $N + P + P + P$ on the left mobile by 20 to get $N + P + 20 = 32$, so $N + P = 12$. Then on the right mobile, $N + P + P + P = 20$, replace $N + P$ by 12 to get $12 + 2P = 20$, so $2P = 8$ and $P = 4$. From the left mobile, you know $N + P = 12$ so $N = 8$.

- ***Two hats and three umbrellas cost $52. Three hats and four umbrellas cost $72. Find the cost of one hat and one umbrella.***

Answer: If H is the cost of a hat and U the cost of an umbrella, then the "mobiles" could be represented by the equations $H + H + U + U + U = 52$ and $H + H + H + U + U + U + U = 72$. Replacing $H + H + U + U + U$ in the left mobile by 52 gives $H + U + 52 = 72$, so $H + U = 20$. Replacing $2H + 2U$ by 40 in the right mobile gives $40 + U = 52$, so $U = 12$ and from the left mobile, $H = 8$, so a hat costs $8 and an umbrella $12.

Class Discussion (continued)

- ***If $a+b=18$ and $2a+b=33$, find a and b.***

  Answer: Create two mobiles where one represents $a+b=18$ and the other $2a+b=33$. Replace $a+b$ in the right "mobile" by 18 so $a+a+b=33$ would be $18+b=33$. $b=15$ and from the left mobile, $a=3$.

- ***If $a+b=20$ and $b=5+2a$, find a and b.***

  Answer: Create two mobiles where one represents $a+b=20$ and the other $b=5+2a$. Replace b in the left mobile by $5+2a$ to get $a+(5+2a)$ to get $3a+5=20$. By the highlight method, $a=5$ and so $b=15$.

Student Activity Questions—Activity 4

1. Let $P=$ the value of the pentagon, *S* the value of the square, *C* the value of the circle, and *T* the value of the triangle. Create the mobiles $2T+P=C+3S$ and $S=2T$.

a. Enter the value of the pentagon as -1 and of the circle as -5. Submit and find the value of the square and the triangle. Explain your thinking, then check to see if you are correct.

Answers will vary. If $P=-1$ and $C=-5$, then the left mobile will be $2T-1=-5+3S$. From the right mobile, $S=2T$, so replacing the $2T$ on the left arm of the left mobile by S, you have a $S-1=-5+3S$. Remove one S from both arms to get $-1=-5+2S$. Using the highlight method, $2S$ has to be 4 so $S=2$, which makes $T=1$.

b. Suppose the pentagon has value $-\frac{1}{2}$, and the circle value has value $-\frac{5}{2}$. Find the values of the square and triangle. Explain your reasoning.

Answer: The square will have value 1 and the triangle $\frac{1}{2}$.The values will be half of the values for question 1a because your given values are half of the given values for question 1a.

c. Create $2T+P=C+S$ and $S=2T$. If $T=-2$, find the values of *C*, *P,* and *S* that will balance the mobiles.

Answer: There are an infinite number of solutions. The mobiles both will be balanced as long as $C=P$.

d. Change one of the two equations in c) so that no values for *C* or *S* will make both of the mobiles balance.

Answers may vary. One strategy is to change the given values and make the pentagon 3 and the circle 5. Because one square is equal to two triangles, the left mobile indicates a pentagon has to equal a circle. The given information makes this impossible so no values for the triangle and the square will make both mobiles balance.

Deeper Dive

- ***On page 2.2, find values for the triangle and the square if the total value for the mobile is 15.***

  Answer: $3T = \frac{15}{2}$ or $7\frac{1}{2}$ so $T = \frac{15}{6}$ which is $2\frac{1}{2}$. $2T + 4S = 7\frac{1}{2}$ so $2\left(\frac{5}{2}\right) + 4S = \frac{15}{2}$; $5 + 4S = \frac{15}{2}$. By the cover up method (with 5 as $\frac{10}{2}$), $4S = \frac{5}{2}$ so $S = \frac{5}{8}$.

- ***Two mobiles are built using four different shapes. If you know the value of one shape, can you find the values of the other three? Explain your thinking.***

  Answer: You may or may not be able to find the value of the other three, but typically, many different values for the other three shapes would make the two mobiles balance. If the shapes are arranged in a contradictory way, you may not be able to find any values for the other shapes, for example, $2S = T + C + P$ and $S = T + C + P$ and the value of $S \neq 0$ is given. If the shapes are arranged exactly the same way or lead to the same relationship among the shapes, there may be an infinite number of values that work, for example, $2S = T + C + P$ and $2S + T = 2T + C + P$.

- ***On page 4.3, create two mobiles using all four shapes. Assign values to two of the shapes so that there is no solution for the values of the other shapes, exactly one solution, and infinitely many solutions for the values of the other shapes.***

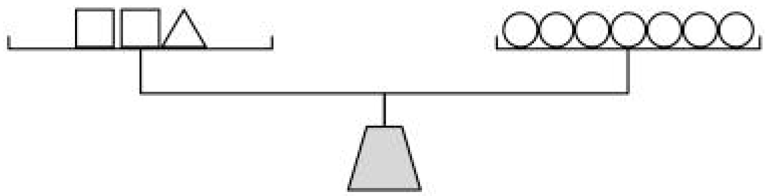
  Answers will vary. Examples might be: no solution $C + T = P$ and $C + T = S$ where $P = 5$ and $S = 8$; one solution $T + C = 2S$ and $C + 2P = 2T$ where $T = 6$, $C = 2$; infinite number of solutions $C = T$ and $P + S = P + S$ where $C = 2$ and $P = 4$.

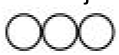
Building Concepts: Visualizing Systems of Linear Equations

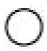
TEACHER NOTES

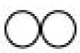
Sample Assessment Items

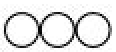
After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.



1. The objects on the scale above make it balance exactly. According to this scale, if  balances , then  balances which of the following?

   a. 

   b. 

   c. 

   d. 

NAEP grade 8 2003

Answer: b

2. A store sells white scarves and red scarves.

   A white scarf costs $3.

   A red scarf costs $5.

   On Monday, the store sold 12 scarves for a total of $50.

   What is the total number of red scarves that the store sold on Monday?

   a. 4

   b. 5

   c. 6

   d. 7

   Answer: d

3. Jamal states that $ax + b = a(x + c)$, given *a*, *b*, and *c* are not equal to 0. What must be the value of *c* for Jamal's statement to be true?

   a. a

   b. b

   c. ac

   d. $\frac{b}{a}$

Smarter Balance, Practice and Training Test, Grade 8. Accessed 9/16/16

Answer: d

4. Mary is buying tickets for a movie.

   Each adult ticket costs $9.

   Each child ticket costs $5.

   Mary spends $110 on tickets.

   Mary buys 14 total tickets.

   Enter the total number of adult tickets and total number of child tickets she buys.

Smarter Balance, Practice and Training Test, Grade 8. Accessed 9/16/16

Answer: She buys 10 adult and 4 child tickets.

5. Simone and Nang read a total of 23 books over the summer. Simone read 5 more books than Nang. How many books Nang did read?

Adapted from Smarter Balance, Practice and Training Test, Grade 8. Accessed 9/16/16

Answer: Nang read 9 books.

Building Concepts: Visualizing Systems of Linear Equations

TEACHER NOTES

Student Activity Solutions

In these activities, you will reason about solutions to systems of equations while modeling the systems on mobiles.

Activity 1 [Page 1.3]

1. Add a circle to the left arm of the mobile.

   a. Set the total value to 36, and determine values for the circle, triangle, and square where at least one of the values is negative**.**

   Answers may vary: $T = 7$, $C = -3$, *and* $S = 1$ *will make both sides 18 for a total value of 36.*

   b. Write an equation representing the relationships between the arms of the mobile. Be sure to define the variables in your equations.

   Answer: If $T =$ *the value of a triangle,* $S =$ *the value of a square, and* $C =$ *the value of a circle, the equation would be:* $3T + C = 2T + 4S$.

   c. Is there a unique (only one) value for each of the shapes represented by your equation in b) that makes the statement true? Why or why not? Give an example to support your reasoning.

   Answer: As long as $T + C = 4S$*, the arms will balance. You can get lots of different values that will make* $T + C = 4S$*, for example,* $S = 2$, $T = 3$, *and* $C = 5$ *or* $S = -6$, $T = -20$, *and* $C = -4$.

Activity 2 [Page 2.2]

1. Use **menu> Mobiles** to select the indicated mobile. Write an equation that represents the relationship among the shapes and find the value of the missing shapes.

   a. **Reset**. Select **Mobile 2**. Let $\text{circle} = -4$ and $\text{square} = 16$. Find the value of a triangle.

   Answer: Equation: $2T + 2C = 2S + T + C$. $\text{triangle} = 36$. *Removing a triangle and a circle from each arm gives the mobile* $T + C = 2S$. *If* $S = 16$ *and* $C = -4$, *then* $T - 4 = 32$, *so* $T = 36$.

   b. **Reset**. Select **Mobile 3**. Let $T = -12$ and $S = 5$.

   Answer: Equation: $2C + 2S + T = 2T + S$. *Taking a square and a triangle from both sides, you get* $2C + S = T$ *or* $2C + 5 = -12$. *By the highlight method,* $C = -\frac{17}{2}$.

   c. **Reset**. Select **Mobile 3**. Replace one triangle on the right arm of Mobile 3 with a circle. Let $C = -4$ and T = 1.

   Answer. Equation: $2C + 2S + T = T + C + S$. *Taking a triangle, a circle, and a square from both sides gives the mobile* $C + S = 0$. *If* $C = -4$, *then* $S = 4$.

Activity 3 [Page 3.2]

1. Determine values for the shapes on the mobile that will balance the mobile and reach the given total value.

   a. **Reset**. Select **menu> Mobiles> mobile 3**. The total value is 48.

   Answer: The only way a triangle, square, and pentagon can make 12 and that a triangle and two squares and a pentagon can make 12 is if a square is 0. Then three squares equals 12, so one square is 4, which makes a pentagon 8. Using those values, $16+3C=12$*, so* $C=-\frac{4}{3}$*. Thus,*

   $S=0$*,* $T=4$*,* $P=8$*, and* $C=-\frac{4}{3}$*.*

   b. Consider the mobile in the previous question. Does it make any difference which of the arms have which of the expressions? Why or why not?

   Answer: It does not make any difference because each of the arms has to have value 12 in order to have a total value of 48.

Activity 4 [Page 4.3]

1. Let $P=$ the value of the pentagon, S the value of the square, C the value of the circle, and T the value of the triangle. Create the mobiles $2T+P=C+3S$ and $S=2T$.

   a. Enter the value of the pentagon as -1 and of the circle as -5. Submit and find the value of the square and the triangle. Explain your thinking, then check to see if you are correct.

   Answers will vary. If $P=-1$ *and* $C=-5$*, then the left mobile will be* $2T-1=-5+3S$*. From the right mobile,* $S=2T$*, so replacing the* $2T$ *on the left arm of the left mobile by S, you have a* $S-1=-5+3S$*. Remove one S from both arms to get* $-1=-5+2S$*. Using the highlight method,* $2S$ *has be 4 so* $S=2$*, which makes* $T=1$*.*

   b**.** Suppose the pentagon has value $-\frac{1}{2}$, and the circle value has value $-\frac{5}{2}$. Find the values of the square and triangle. Explain your reasoning.

   Answer: The square will have value 1 and the triangle $\frac{1}{2}$*.The values will be half of the values for question 1a because your given values are half of the given values for question 1a*.

   c. Create $2T+P=C+S$ and $S=2T$. If $T=-2$, find the values of C, P, and S that will balance the mobiles**.**

   Answer: There are an infinite number of solutions. The mobiles will both be balanced as long as $C=P$*.*

d. Change one of the two equations in c) so that no values for *C* or *S* will make both of the mobiles balance.

Answers may vary. One strategy is to change the given values and make the pentagon 3 and the circle 5. Because a square is equal to two triangles, the left mobile indicates a pentagon has to equal a circle. The given information makes this impossible, so no values for the triangle and the square will make both mobiles balance.