Building Concepts: Connecting Ratios to Graphs

Lesson Overview

In this TI-Nspire™ lesson, students consider the values in a ratio table as ordered pairs and graph them on coordinate axes. Students learn that the graph of a collection of equivalent ratios lies on a line through the origin.

The graph of a collection of equivalent ratios lies on a line through the origin.

Learning Goals

1. Interpret ratios as ordered pairs and plot the points associated with a ratio;
2. recognize that the graph of a collection of equivalent ratios lies on a ray through the origin;
3. relate a table of ratios to a graph, including explaining the connections in terms of repeated addition or scalar multiplication;
4. identify patterns in the graphs of the points associated with a ratio.

Prerequisite Knowledge

Connecting Ratios to Graphs is the eighth lesson in a series of lessons that explore the concepts of ratios and proportional relationships. This lesson builds on students' prior knowledge of plotting points in a coordinate grid by associating a ratio with an ordered pair of values. Prior to working on this lesson, students should have completed *Ratio Tables*, *Comparing Ratios*, and *Ratios and Fractions.* Students should understand:

- the concept of ordered pairs;
- how to plot points on a coordinate grid;
- how to complete a ratio table.

Vocabulary

- **ordered pair**: a pair of numbers used to locate a point on a coordinate plane where the first number in the pair represents the horizontal distance from the origin and the second number in the pair represents the vertical distance from the origin.

Lesson Pacing

This lesson contains multiple parts and can likely be completed in 2–3 class periods, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Connecting Ratios to Graphs_Student.pdf
- Connecting Ratios to Graphs_Student.doc
- Connecting Ratios to Graphs.tns
- Connecting Ratios to Graphs_Teacher Notes
- To download the TI-Nspire lesson (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS lesson as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.

Additional Discussion: These questions are provided for additional student practice and to faciliate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

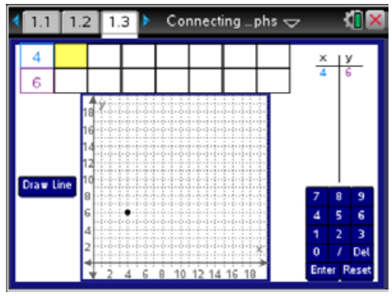
In earlier lessons, students generated tables of equivalent ratios. In this TI-Nspire™ lesson, students consider the values in a ratio table as ordered pairs and graph them on coordinate axes. The graph of a collection of equivalent ratios lies on a line through the origin. Careful inspection of tables containing equivalent ratios illustrates regularity in the entries and the corresponding graphs, which can be explained in terms of repeated addition or scalar multiplication. The pattern of change in the table can be seen in the graph as coordinated horizontal and vertical steps used when plotting a point in the coordinate grid. Some students struggle in later grades with distinguishing between the plot of a point such as (2, 3) and how those values are related to an equation of a line through the point. Emphasize the connection between "for every 2 over to the right, go up 3" when moving from point to point using an additive strategy and the ordered pairs ($2a$, $3a$) that represent all the ratios equivalent to 2:3 from a multiplicative perspective. This can help students make sense of what the ordered pairs represent and how to plot them. Note that the language "rise over run" is not used, as it can be misleading and confusing, particularly when dealing with integers. Students might want to move vertically, then horizontally to plot a point, which produces the same result; moving horizontally first then vertically supports thinking about the horizontal axis as representing the independent variable and the vertical axis as the dependent variable.

Connecting the point (2, 3) to the point (4, 6) by thinking "for every 2 over go up 3" can also be extended to consider the relationship between the point (2, 3) and the point (6, 9), which would be "for every 6 over go up 9." This gives students experience in visualizing equivalent ratios and interpreting them in different contexts. Interpreting the patterns in the ratio table and in the graph of equivalent ratios introduces students to the notion of rate of change—or slope and slope triangles—as well as to similar triangles.

The number 0 cannot be used in a ratio. For example, 0:0 does not make sense because equivalent ratios are generated by multiplying or dividing each value in a ratio by the same positive number.

Part 1, Page 1.3

Focus: What patterns occur when a collection of equivalent ratios is graphed in a coordinate plane?



In Part 1, students connect a ratio to an ordered pair that can be graphed in a coordinate plane and a set of equivalent ratios to points that lie on a ray through the origin. They relate a ratio table to a “t-table” to the points on the graph.

On page 1.3, selecting a cell in either row will highlight the cell. Students can use the keypad to specify any positive whole number value in the highlighted cell. Students can change the original ratio in the horizontal table by selecting each of the cells in the first column.

Enter a positive whole number in the first cell in the top and bottom row of the table to set the ratio. Continue entering positive whole numbers to generate a table of equivalent ratios. Use the arrow keys to move the cursor in the table. When 0 is entered into one of the cells, both values are greyed out because 0:0 is not a ratio.

TI-Nspire Technology Tips

The enter key fills in the missing value for the equivalent ratio in both tables and shows the point plotted on the grid.

To draw the line, select the **Draw Line** button on the screen

To hide the line, select **Hide Line** button on the screen.

Reset returns to the original screen or press ctrl del to reset.

Teacher Tip: After students enter a value in the first cell of the column have them predict what the second number will be. Encourage students to identify the patterns they see in the table.

Class Discussion

- ***Describe the connection between the ratio 4:6 and the point plotted on the grid.***

  Answer: The point is the ordered pair (4, 6), which corresponds to the ratio 4:6.

- ***Highlight a cell in the top row of the table and type in a multiple of 2, then select*** enter. ***Highlight a cell in the bottom row and type in a multiple of 3, and then select*** enter. ***What can you say about the values that appear in the columns?***

  Answer: The ordered pair (9, 6) means that to line up nine students, you will need 6 tiles.

- ***Each ratio has been associated with a point in the graph. Identify two ordered pairs displayed in the graph and explain how they were plotted.***

  Answers will vary. Possible ordered pairs are: (2, 3) is located at 2 on the horizontal axis and 3 on the vertical axis, (4, 6) is located at 4 on the horizontal axis and 6 on the vertical axis, (6, 9) is over to the right 6 and up 9 from the origin (0, 0).

- ***Find three other ratios equivalent to 4:6 and enter them into the table. What observation can you make about the graph of the five ratios?***

  Ratios will vary, but the points should all lie on a straight line.

- ***Which of the following points will also follow the same pattern even if they do not appear in the table? Explain your thinking.***

  a. $(1, \frac{2}{3})$

  b. $(1, \frac{3}{2})$

  c. *(24, 36)*

  d. *(3, 2)*

  Answer: To be on the line, the ratios associated with the points have to be equivalent to 2:3. Only b and c are equivalent (multiply the values in 1:$\frac{3}{2}$ by 2 and divide the values in 24:36 by 12 to get 2:3).

Reset the page and use the keypad to enter the ratio 1:4 in the first column in the table at the top of the screen.

- ***Explain how to plot the point associated with 1:4***.

  Answer: The point is 1 over to the right on the horizontal axis and 4 up on the vertical axis from the point (0, 0).

- ***Find five equivalent ratios and enter them into the table. Use the Draw Line command. How do the points seem to be related?***

  Answer. The points should be on the same straight line.

Class Discussion (continued)

- ***What value for x will make the ratio x:3 equivalent to 1:4? Explain how you found your answer and how you know your answer is correct.***

  Answer: x = $\frac{3}{4}$ = 0.75. Students could enter 3 into the second column of the table, recognize that $\frac{3}{4}$ times each value in 1:4 will give 3 as the second value in the ratio pair, or they could draw a line and read the ordered pair from the graph.

Student Activity Questions—Activity 1

1. **2 pounds of fruit cost $5. Reset the ratio to 2:5 and draw the line on the graph.**

   a. **Find a point on the line and explain what it means in terms of the amount of fruit.**

   Answers will vary. For example, (4, 10) would mean 4 pounds of fruit cost $10.

   b. **Enter one of the coordinates of your point from part a in the table, and then select enter. How do the values in the table associated with the point relate to the other values in the table?**

   Answers may vary. Some might note that the coordinates of the new point are some multiple of the original ratio (twice each of the original values for the example above). Others might read down the table, for example, the *x*-coordinate is two more and the *y*-coordinate is 5 more.

   c. **Sammy argued that 6 pounds of fruit should cost $9 because 4 more pounds of fruit should cost $4 more. Do you agree or disagree with Sammy? Find at least two different ways to explain your thinking.**

   Answer: Sammy is wrong because to keep the same ratio, you need 2 pounds for every $5. Adding two 2 pounds would require you to add 2 times $5, or $10; so adding 4 pounds should cost more than $9. A second way to reason is that to have an equivalent ratio, you need to multiply or divide both values in the ratio by the same positive number. To go from 2 pounds to 6 pounds, you multiply by 3, so the price should be multiplied by 3, which is $15 for 6 pounds. A third strategy might be to figure out (either from the line or by dividing) that the unit price is really $1.50 per pound. So for 6 pounds you would need 6 × $1.50 or $15.

   d. **Find at least three equivalent ratios such that at least one of the values in each of them is not a whole number. Explain how you found your answers and describe what each means in terms of the context.**

   Answers will vary. Sample responses: (1, $\frac{5}{2}$) would mean 1 pound of fruit for $2.50; (3, $\frac{15}{2}$) would be 3 pounds of fruit for $7.50; and ($\frac{4}{5}$, 2) would mean 0.8 of a pound for $2.

Additional Discussion

This is a contextual problem that can be analyzed using the graph.

Reset then change the values in the first column in the table to the new ratio. With a partner, predict the values in three columns in the table and predict three points you think will be on the graph for each ratio.

- **2:1**

  Answers will vary. Possible answers might be 4:2; 6:3; 8:4 and the points (4, 2), (6, 3), (8, 4).

- **1:2**

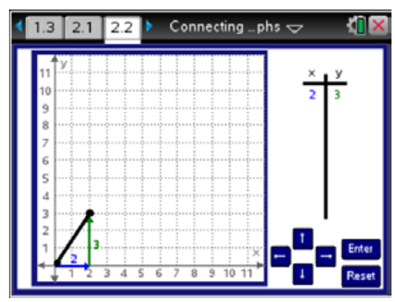
  Answers will vary. Possible answers might be 2:4; 3:6; 4:8 and the points (2, 4), (3, 6), (4, 8).

- ***Have one partner check the predictions for question 3a, 2:1, and the other check the predictions for question 3b, 1:2. How do the two graphs compare?***

  Answer: The line that would contain the points related to the ratio 2:1 is steeper than the line that would contain the points for the ratio 1:2.

Part 2, Page 2.2

Focus: Developing an additive strategy for creating points representing equivalent ratios of the form ***a:b***, where students visualize moving from a given point first horizontally by ***a*** units and then vertically by ***b*** units to generate a second point in a collection of equivalent ratios. This informally introduces the notion of a "slope triangle".



Select the right arrow once and the up arrow once to generate the next point in the collection of equivalent ratios. To generate another point, use the new horizontal arrow. Move the point to set a new starting point.

Set the rate by:

- Dragging the endpoint of the segment; or
- Using the arrow keys to move the endpoint of the segment. Press enter to set the slope of the line.

Build the line by using the right and up arrows on the keyboard or on the screen.

> **Teacher Tip:** Guide students in a discussion of the relationship between the entries in the t-table. Have students explain how that relationship is shown on the coordinate grid. Encourage students to explore the lesson with other equivalent ratios.

Class Discussion

Have students…	Look for/Listen for…
• ***Reset the page. What ratio is represented by the point in the grid?***	Answer: The ordered pair (2, 3) represents the ratio 2:3.
• ***Using the arrows at the lower left on the grid, select the blue right arrow once and up arrow once. Describe how what you see on the screen is related to a collection of equivalent ratios.***	Answer: A new point on the line is (4, 6), which is over 2 and up 3 from the original point (2, 3). The coordinate (4,6) is associated with the ratio 4:6, which is equivalent to the ratio 2:3.
• ***Predict what you think will happen if you select the blue horizontal and vertical arrows again. Explain your reasoning.***	Answers will vary. Possibly (6, 9) because the next point will again be over 2 from 4 and up 3 from 6.
• ***Suppose you were able to select the horizontal and vertical blue arrows enough times to display six points on the line. What would be the coordinates of the sixth point? Explain how you know.***	Answer. Every time you select the blue arrows, you go over 2 and up 3, so the point would be at (10, 15), which would also be the table value.

Student Activity Questions—Activity 2

1. **Reset. Move the point to (2, 1). Generate four more points on the graph. Explain the correspondence between the table and the graph.**

   Answer: The table shows points associated with the ratios equivalent to 2:1. The graph shows how you can get from one point to the other, by going over 2 and up 1. You can see the same thing in the table as you move down the table; each *x*-value is 2 more than the previous value, and each *y*-value is 1 more than the previous *y*-value.

2. **Move the point to (4, 3).**

   a. **If you select the right arrow once and the up arrow once, what will the next point on the line be? Check your answer using the TNS lesson.**

      Answer: over 4 and up 3 or at (8, 6)

   b. **Identify two points you think will be on the line that you cannot generate using the horizontal and vertical arrows. Explain why you think those points will be on the line.**

      Possible answer: Use the same ratio of 4 over to 3 up to generate the next two points from (8, 6), (12, 9), and (16, 12). Students might reason from the table that each successive row is generated by adding 4 to the *x*-value and 3 to the *y*-value in the previous row. Others might argue that points should be on the line because each point is a multiple of the values in the ratio 4:3, 3 times the values to get (12, 9) and 4 times the values to get (16, 12).

Student Activity Questions—Activity 2 (continued)

3. **Identify at least three points on each using the following instructions. Assume for each that you begin at (0, 0). The TNS lesson might help with thinking about the problem or checking your work.**

   a. **moving over 2 units and up 1 unit.**

   Answers may vary: Possible responses include (2, 1); (4, 2); (6, 3); (8, 4).

   b. **moving over 2 units and up 4 units.**

   Answers may vary: Possible responses include (2, 4); (4, 8); (6, 12); (8, 16).

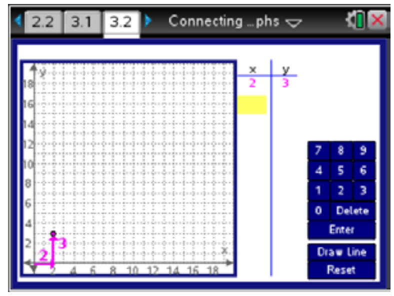
   c. **moving over 1 unit and up 5 units.**

   Answers may vary: Possible responses include (1, 5); (2, 10); (3, 15); (4, 20).

Additional Discussion

Have students…	Look for/Listen for…
Explain your reasoning in each case.	
If the ratio is 4:7, which of the following will always be true about the rate of change in moving from one point to another on the line?	
• ***moving over 4 units and up 7 units***	Answer: True because the ratio 4:7 can be associated with the ordered pair (4, 7), which is over 4 and up 7 from the origin.
• ***moving over 7 units and up 4 units***	Answer: False because this would be the point (7, 4), which is not associated with a ratio equivalent to 4:7
• ***moving over 1 unit and up*** $1\frac{3}{4}$ ***of a unit***	Answer: True because the ratio 1: $1\frac{3}{4}$ can be multiplied by 4 to get the original ratio of 4:7.
• ***moving up 2 units and over 3.5 units***	Answer: True because the ratio 2:3.5 is equivalent to $\frac{1}{2}$ of each value in 4:7.

Part 3, Page 3.2



Focus: Developing a multiplicative strategy for creating points representing equivalent ratios. The points do not have to be plotted in a sequential order, and the visual image of plotting points representing two equivalent ratios lays the groundwork for scaling figures and for more formal work with similarity.

Selecting a cell in the t-table highlights the cell. The keypad can be used to type a number in the cell. Using enter fills in the corresponding cell in the row and displays the horizontal and vertical moves to locate the point on the grid.

Draw Line displays the line through the points. The graph display has been restricted on this page, so points may appear on the table, but not on the graph.

Build the table by entering numbers in the *x* or *y* columns in the table.

To draw the line, press the **Draw Line** button on the screen. To hide the line, press the **Hide Line** button on the screen. The menu key on the handheld can also be used to Draw/Hide the line.

Class Discussion

Type 8 in the cell below 2, and then select enter. Draw line.

Have students…	**Look for/Listen for…**
• ***Explain how the table and the graph correspond.***	Answer: The graph shows how far over and how far up the change would be to move from (0, 0) to the point (8, 12) on the line determined by the ratios equivalent to 2:3. The table shows the coordinates of the original point (2, 3) and of the new point (8, 12).
• ***What is the x-value if the y-value is 15? Check your answer by typing a 15 in the third cell in the second column and selecting enter.***	Answer: The *x*-value will be 10.

Class Discussion (continued)

- ***How is the table for this set of ordered pairs the same as the table on page 2.2? How is it different?***

Answer: Both tables show the coordinates associated with values of ratios equivalent to 2:3, but on page 2.2, there is a pattern or regularity in the *x*-values, 2 more than the *x*-value in the previous row, and in the *y*-values, 3 more than the *y*-value in the previous row. The values in the table on page 3.2 are multiples of the original coordinates but do not have the same regularity as the values in page 2.2's table.

- ***Compare how the lines are generated on page 2.2 and 3.2***

Answer: The graph on page 2.2 shows a "triangle" going over and up from each point in the table to the next point. All of the triangles have the base 2 and the height 3. The graph on page 3.2 shows just two triangles, the original one with base 2 and height 3 and then a triangle associated with the last point in the table, with the horizontal distance matching the *x*-value in the table and the vertical distance the *y*-value in the table.

Reset, then enter the values for (4, 3) in the top row of the table.

- ***Predict the y-value for an x-value of 16. Then, predict the x-value for a y-value of 9. Check your answer using the TNS lesson.***

Answer: The *y*-value will be 12 for an *x*-value of 16; the *x*-value will be 12 for a *y*-value of 9.

- ***If you were to enter x = 1 in the first column, what would you expect the corresponding y-value in the second column to be? Explain your reasoning, and then check your answer using the TNS lesson.***

Answer: $\frac{3}{4}$ because multiplying the values in the ratio 4:3 by $\frac{1}{4}$ (or dividing them by 4) will produce the ratio $1:\frac{3}{4}$, which will be associated with the point $(1, \frac{3}{4})$.

- ***Draw the line. Find the horizontal and vertical component of another point that is on the line but off the grid.***

Possible answer: a horizontal component of 24 and a vertical component of 18, for the point (24, 18).

Class Discussion (continued)

Set the TNS page to display a ratio of 12 pieces of pizza for 18 people. Which of the following points will be on the line determined by the ratio? Explain what each point means in terms of the pizza.

- ***a.*** ***(2, 3)***
- ***b.*** ***(8, 12)***
- ***c.*** ***(10,16)***
- ***d.*** $\left(\frac{2}{3}, 1\right)$

Answer: All of the points except c will be on the line: a) is 2 pieces of pizza for every 3 people, b) is 8 pieces of pizza for every 12 people, and d) is $\frac{2}{3}$ of a piece of pizza per person.

Student Activity Questions—Activity 3

1. Set up the TNS page for the following ratio: 2 centimeters on a blueprint represent 7 meters.

a. Create a table that would help the blueprint maker convert at least six dimensions from the blueprint into meters. Explain how you created your table.

Answers may vary. Sample table entries are below. Students might find the table by adding or subtracting 7 meters for every more 2 centimeters more in length, by using a rate of 3.5 meters for every centimeter, by multiplying a given number of centimeters by 3.5, or by finding multiples of the ratio 2:7.

centimeters	**meters**
1	3.5 $\left(\frac{7}{2}\right)$
2	7
3	10.5 $\left(\frac{21}{2}\right)$
4	14
5	17.5 $\left(\frac{35}{2}\right)$
6	21
10	35

b. A room is 35 meters wide. How long would it be on the blueprint?

Answer: 10 centimeters

Student Activity Questions—Activity 3 (continued)

c. **Geoff claims that every centimeter on the blueprint represents $3\frac{1}{2}$ meters. Do you agree? Why or why not?**

Answer: Yes, because 2 cm for every 7 m is equivalent to 1 cm for every 3.5 m.

2. **Use the TNS lesson to help answer the following questions. Sari wants to make salad dressing using a recipe that calls for 2 tablespoons of vinegar and 5 tablespoons of oil.**

a. **Suppose she wants to keep the same ratio of vinegar to oil. How much oil should she use for 1 tablespoon of vinegar?**

Answer: $2\frac{1}{2}$ or 2.5 tablespoons

b. **Sari claimed that for 15 tablespoons of oil, she needed 9 tablespoons of vinegar. How can you use the graph to decide whether she was right or not?**

Answer: The line does not go through the point (9, 15). It should be (6, 15) or 6 tablespoons of vinegar for 15 of oil.

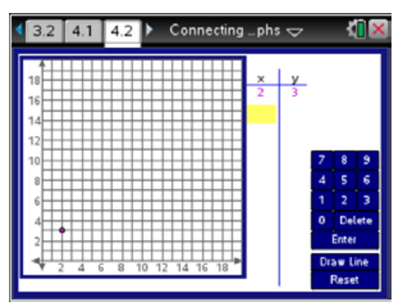
c. **If Sari added 2 more tablespoons of vinegar, how much more oil should she add to keep the same ratio?**

Answer: 2 more tablespoons of vinegar would make a total of 4 tablespoons of vinegar. So, to keep the same ratio of 2 vinegar to 5 oil, she would need to have added 5 tablespoons of oil to get the ratio 4 vinegar to 10 oil.

d. **If Sari accidentally used 7 tablespoons of oil, how much vinegar should she use to keep the same ratio?**

Answer: 2.8 tablespoons of vinegar

Part 4, Page 4.2



Part 4 is optional. Included are contextual problems involving numbers larger than 20. You may want to work with students through page 4.2 or have students work the problems first, and then use the lesson to check their results.

This page works the same as page 3.2, except you can change the scale of the grid.

Press the menu key or and select “Grid Scale” or “Grid Max” to change the scale for your graph.

Teacher Tip: Guide students in a discussion about choosing the appropriate scale for the grid axes. Ask them to think about how they might choose the right scale on the graph and why they might want to consider this as a constraint for their problems. Encourage students to explore using grids with different scales for the same table.

Class Discussion

Have students…	Look for/Listen for…
Reset the page. The school band has 4 boys to every 5 girls. Enter the ratio in the table and draw the line.	
• ***Use the TNS lesson to plot the point representing the number of boys and girls in the band if there are 15 girls. How is this point related to the original point?***	Answer: Using the ratio of boys to girls as 4:5, the point would be (12, 15) and would represent 12 boys and 15 girls. The point represents a ratio that is equivalent to the original ratio. (Note some students may use the ratio 5:4, which will yield the same answers but from a different ratio and a different line. Allow for some discussion about why the different ratios may give the same result.)
• ***Suppose there are over 40 students in band. Find three possible numbers of boys and girls in the band.***	Answers may vary: Possible responses include 20 boys and 25 girls, 24 boys and 30 girls, and 28 boys and 35 girls.

Additional Discussion

Have students…	Look for/Listen for…
• ***How are the ordered pairs you found in the question above related to the original ratio?***	Answer: They are all on the same line and represent ratios equivalent to 4:5.
Simon usually runs at a steady pace of 5 meters every 2 seconds. Use the TNS lesson to help you answer each question.	
• ***If he triples the time he runs, will he triple the distance? Why or why not?***	Answer: Yes, as long as he runs at the same pace because the ratio of 5 meters in 2 seconds is equivalent to 15 meters in 6 seconds.
• ***If he adds 6 seconds to the time he runs, will he add 6 meters to the distance he runs? Why or why not?***	Answer: No, because the ratio 5 meters in 2 seconds is not equivalent to 11 meters in 8 seconds.
• ***If he runs for a minute, how far will he run?***	Answer: 150 meters.

Additional Discussion (continued)

- ***If he runs 100 meters, how long will it take him?***

Answer: 40 seconds

Simon walks at a rate of 3 meters every 2 seconds. Which of the following describes his pace? You may want to use the TNS lesson to help your thinking.

a. 1 meter every $\frac{2}{3}$ second

b. 9 meters every 6 seconds

c. 1.5 meters per second

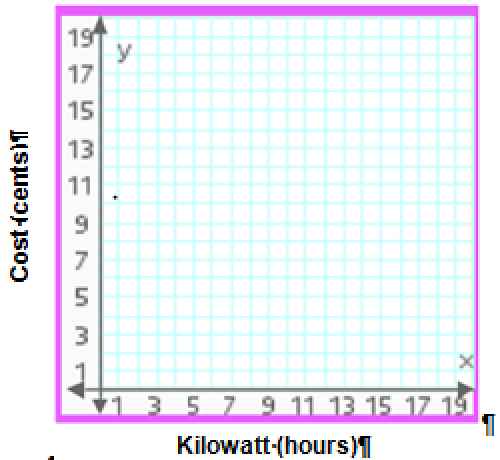
d. 45 meters every half a minute

Answer: All of the choices are correct. The ratios in each case are equivalent to 3:2; for 1: $\frac{2}{3}$, multiply the values by 3; for 9:6, divide the values by 3; for 1.5:1, multiply the values by 2; for 45:30, divide the values by 15.

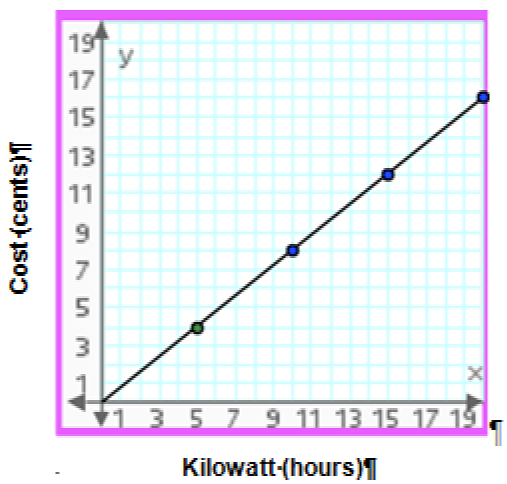
Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS lesson.

1. Solar energy in one region costs 4 cents for 5 kilowatt hours usage. Make a graph that shows how much it would cost for using up to 20 kilowatt hours of energy. Identify at least three points in your graph.

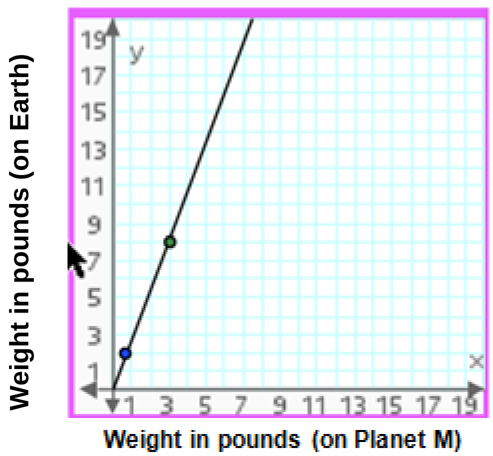


Sample Answer:



Sample Assessment Items (continued)

2. An object on Earth is heavier than the same object on Planet M. The graph shows the relationship between the two weights.



   a. What is the ratio of the weight on Earth to the weight on Planet M? ***Answer: 3 pounds on Planet M for every 8 pounds on Earth***

   b. If an object on Earth weighs 16 pounds, how much will it weigh on Planet M? ***Answer: 6 pounds***

   c. If an object on Planet M weighs 45 pounds, how much will it weigh on Earth? ***Answer: 120 pounds***

3. A game is set up so a player scores 7 points for every 2 hits, represented by the point (7, 2). Which of the following are possible outcomes in the game?

   a. (28, 8) b. (3.5, 1) c. (9, 4) d. (14, 49)

   Answer: a) (28, 8) and b) (3.5, 1)

4. Which of the following points will lie on the line that contains points associated with the collection of ratios equivalent to 8:5?

   a. (10, 16) b. (2, 1.25) c. (24, 15) d. (14, 11)

   Answer: b) (2, 1.25) and c) (24, 15)

Building Concepts: Connecting Ratios to Graphs

Student Activity Solutions

In these activities you will work together to identify patterns in the graphs of the points associated with a ratio and use them to solve problems. After completing each activity, discuss and/or present your findings to the rest of the class.

Activity 1 [Page 1.3]

1. 2 pounds of fruit cost $5. Reset the ratio to 2:5 and draw the line on the graph.

   a. Find a point on the line and explain what it means in terms of the amount of fruit.

   Answers will vary. For example, (4,10) would mean 4 pounds of fruit cost $10.

   b. Enter one of the coordinates of your point from part a in the table, and then select enter. How do the values in the table associated with the point relate to the other values in the table?

   Answers may vary. Some might note that the coordinates of the new point are some multiple of the original ratio (twice each of the original values for the example above). Others might read down the table, for example, the x-coordinate is two more and the y-coordinate is 5 more.

   c. Sammy argued that 6 pounds of fruit should cost $9 because 4 more pounds of fruit should cost $4 more. Do you agree or disagree with Sammy? Find at least two different ways to explain your thinking.

   Answer: Sammy is wrong because to keep the same ratio, you need 2 pounds for every $5. Adding two 2 pounds would require you to add 2 times $5, or $10; so adding 4 pounds should cost more than $9. A second way to reason is that to have an equivalent ratio, you need to multiply or divide both values in the ratio by the same positive number. To go from 2 pounds to 6 pounds, you multiply by 3, so the price should be multiplied by 3, which is $15 for 6 pounds. A third strategy might be to figure out (either from the line or by dividing) that the unit price is really $1.50 per pound. So for 6 pounds you would need 6 $\times$ $1.50 or $15.

   d. Find at least three equivalent ratios such that at least one of the values in each of them is not a whole number. Explain how you found your answers and describe what each means in terms of the context.

   Answers will vary. Sample responses: (1, $\frac{5}{2}$) would mean 1 pound of fruit for $2.50; (3, $\frac{15}{2}$) would be 3 pounds of fruit for $7.50; and ($\frac{4}{5}$, 2) would mean 0.8 of a pound for $2.

Activity 2 [Page 2.2]

1. Reset. Move the point to (2, 1). Generate four more points on the graph. Explain the correspondence between the table and the graph.

   Answer: The table shows points associated with the ratios equivalent to 2:1. The graph shows how you can get from one point to the other, by going over 2 and up 1. You can see the same thing in the table as you move down the table; each x-value is 2 more than the previous value, and each y-value is 1 more than the previous y-value.

2. Move the point to (4, 3).

   a. If you select the blue horizontal arrow twice, what will the next point on the line be? Check your answer using the TNS lesson.

   Answer: over 4 and up 3 or at (8, 6)

   b. Identify two points you think will be on the line that you cannot generate by selecting the blue horizontal arrow. Explain why you think those points will be on the line.

   Possible answer: Use the same ratio of 4 over to 3 up to generate the next two points from (8, 6), (12, 9), and (16, 12). Students might reason from the table that each successive row is generated by adding 4 to the x-value and 3 to the y-value in the previous row. Others might argue that points should be on the line because each point is a multiple of the values in the ratio 4:3, 3 times the values to get (12, 9) and 4 times the values to get (16, 12).

3. Identify at least three points on each using the following instructions Assume for each that you begin at (0, 0). The TNS lesson might help with thinking about the problem or checking your work.

   a. moving over 2 units and up 1 unit.

   Answers may vary: Possible responses include (2, 1); (4, 2); (6, 3); (8, 4).

   b. moving over 2 units and up 4 units.

   Answers may vary: Possible responses include (2, 4); (4, 8); (6, 12); (8, 16).

   c. moving over 1 unit and up 5 units.

   Answers may vary: Possible responses include (1, 5); (2, 10); (3, 15); (4, 20).

Activity 3 [Page 3.2]

1. Set up the TNS page for the following ratio: 2 centimeters on a blueprint represent 7 meters.

   a. Create a table that would help the blueprint maker convert at least six dimensions from the blueprint into meters. Explain how you created your table.

   Answers may vary. Sample table entries are below. Students might find the table by adding or subtracting 7 meters for every more 2 centimeters more in length, by using a rate of 3.5 meters for every centimeter, by multiplying a given number of centimeters by 3.5, or by finding multiples of the ratio 2:7.

centimeters	meters
1	*3.5* $\left(\frac{7}{2}\right)$
2	*7*
3	*10.5* $\left(\frac{21}{2}\right)$
4	*14*
5	*17.5* $\left(\frac{35}{2}\right)$
6	*21*
10	*35*

   b. A room is 35 meters wide. How long would it be on the blueprint?

   Answer: 10 centimeters

   c. Geoff claims that every centimeter on the blueprint represents $3\frac{1}{2}$ meters. Do you agree? Why or why not?

   Answer: Yes, because 2 cm for every 7 m is equivalent to 1 cm for every 3.5 m.

2. Use the TNS lesson to help answer the following questions. Sari wants to make salad dressing using a recipe that calls for 2 tablespoons of vinegar and 5 tablespoons of oil.

    a. Suppose she wants to keep the same ratio of vinegar to oil. How much oil should she use for 1 tablespoon of vinegar?

    Answer: $2\frac{1}{2}$ *or 2.5 tablespoons*

    b. Sari claimed that for 15 tablespoons of oil, she needed 9 tablespoons of vinegar. How can you use the graph to decide whether she was right or not?

    Answer: The line does not go through the point (9, 15). It should be (6, 15) or 6 tablespoons of vinegar for 15 of oil.

    c. If Sari added 2 more tablespoons of vinegar, how much more oil should she add to keep the same ratio?

    Answer: 2 more tablespoons of vinegar would make a total of 4 tablespoons of vinegar. So, to keep the same ratio of 2 vinegar to 5 oil, she would need to have added 5 tablespoons of oil to get the ratio 4 vinegar to 10 oil.

    d. If Sari accidentally used 7 tablespoons of oil, how much vinegar should she use to keep the same ratio?

    Answer: 2.8 tablespoons of vinegar