Building Concepts: Outliers

TEACHER NOTES

Lesson Overview

In this TI-Nspire lesson, students will develop a working definition of outliers and use it to analyze data sets. Students will use multiples of $\frac{1}{2}$ IQR to define the boundaries for points to be considered outliers. Finally, students will recognize the effects of outliers on measures of center and spread. In several cases students are encouraged to think of plausible explanations for outliers in the context of the data.

Outliers are extreme data values, worth noting for contextual reasons and for their effects on center and spread.

Learning Goals

1. Use the formula for outliers to identify outliers in a set of data;
2. describe the impact of outliers on measures of center and spread;
3. visually identify outliers in a set of data.

Prerequisite Knowledge

Outliers is the eighth lesson in a series of lessons that investigates the statistical process. In this lesson, students use outliers to analyze data sets. This lesson builds on the concepts of the previous lessons. Prior to working on this lesson students should have completed *Median and Interquartile* Range and *Tables and Measures*. Students should understand:

- how identify lower, upper, and interquartile ranges;
- how to interpret data on a box plot.

Vocabulary

- **outlier:**. any value that lies outside of the box by a distance of $\frac{3}{2}$ IQR in either direction
- **upper quartile:** the median of the *upper* half of a data set
- **lower quartile:** the median of the *lower* half of a data set
- **interquartile range:** the difference between the upper quartile and the lower quartile

Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software
- Outliers_Student.pdf
- Outliers_Student.doc
- Outliers.tns
- Outliers_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.

Deeper Dive: These questions are provided for additional student practice and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

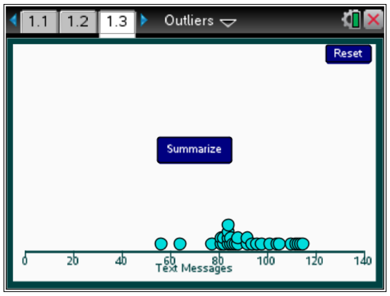
Mathematical Background

Distributions of data are characterized by their shape, center and spread. In some cases, the distribution includes values that do not seem to fit the pattern of the other data. Extreme values, far from the other values, are called *outliers*: in this lesson, an outlier is any value that is more than 1.5 times the interquartile range above the upper quartile or less than 1.5 times the interquartile range below the lower quartile, i.e., an outlier is a value x such that $x > \text{UQ}+1.5 \cdot \text{IQR}$ or $x < \text{LQ} - 1.5 \cdot \text{IQR}$. John Tukey (1984), who originated exploratory data analysis, when asked about why $1.5 \cdot \text{IQR}$ was used responded that one IQR identified too many outliers and two IQRs did not identify enough. Other formulas used for outliers typically involve the mean and some measure of spread; these are considered in more advanced courses.

Outliers can be important signals about the data. Outliers may occur because of processing errors (mistyping, transposing digits, etc.); some outliers indicate some aspect of the data that might be useful in understanding the story in the data such as indicating a possible new trend (for example, a different medicine made a great improvement in a patient or a spike in the number of crimes in a city might signal the need to rethink how the city is patrolled).

Part 1, Page 1.3

Focus: Students identify outliers using the measure $1.5 \cdot \text{IQR}$.



On page 1.3, the data shown are the number of text messages sent or received in a day by students in one class.

Summarize shows a box plot of the data.

$\mathbf{UQ - LQ = IQR}$ shows the IQR with a vertical dotted segment marking $\frac{1}{2}$ of the width of the box and removes the segments at the ends of the box.

Move boundary out 1/2 IQR extends dotted lines in multiples of $\frac{1}{2}$ IQR beyond the box.

Box plots/Outliers shows a box plot with outliers defined by the boundaries.

Points are moveable by dragging.

TI-Nspire Technology Tips

menu accesses page options.

tab cycles through the points or the on screen buttons.

Up/Down arrows control what tab access.

Left/Right arrows move selected points.

esc releases all selected points.

ctrl del resets the page.

Class Discussion

In the following questions students explore moving a length of $\frac{1}{2}$ IQR from each end of the box that represents $\mathbf{UQ - LQ = IQR}$ in a box plot. They should observe the numerical length for a given IQR and be able to calculate the lengths for different multiples of $\frac{1}{2}$ IQRs.

Page 1.3 displays a dot plot of the number of text messages sent or received in a day by students in one class.

- ***What does the point at about 79 mean?***

  Answer: One student sent or received 79 text messages a day.

- ***What might strike you as unusual about the distribution of text messages?***

  Answers may vary. Some students might point out that two of the students seemed to send/receive fewer messages that the rest of the class.

Class Discussion (continued)

- ***Describe the distribution of the number of text messages sent or received.***

  Answers may vary. Except for two students who send/receive around 60 messages a day, the distribution is skewed right. Most of the students send/receive between about 78 and 90 messages a day with some going to nearly 120.

- ***Estimate the median and the IQR and explain what each means.***

  Answers will vary. Students might suggest 82 messages for the median, which indicates that half of the students in the class sent/received 82 or more messages a day, while half sent/received 82 or less. An estimate of the IQR is about 20, which indicates that spread of the number of messages sent by the middle half of the students is only 20; students are pretty alike in how many messages they send/receive.

Select **Summarize**.

- ***Why is the segment on the box plot to the left longer than the segment to the right?***

  Answer: The number of messages students in the lower fourth of the distribution send/receive is spread out from below 60 to about 85 messages, while the number of messages students in the top fourth of the distribution send/receive is spread over a shorter span, from about 102 to 118 messages.

- ***Select*** $\mathbf{UQ - LQ = IQR}$***. Describe what changed.***

  Answer: The segments disappeared and left the box that shows the length of the IQR. The median disappeared and a vertical segment showed up in the middle of the box as well as a dotted horizontal segment through the box. The IQR shows on the screen. The dots that are not represented in the box are colored orange.

- ***Select*** **Move boundary out 1/2 IQR*****. Describe what changed.***

  Answer: The dotted line in the box is gone, and dotted horizontal segments are outside the box on both ends. Now the dots representing students outside of the dotted segments are colored orange. All those inside the box and dotted segments are blue/green.

- ***What do the dotted segments represent and how long are they?***

  Answer: They represent the length $\frac{1}{2}$ IQR, which is 8.5.

Class Discussion (continued)

Move out until none of the dots are orange.

- ***How many IQRs did you move on either side of the box?***

  Answer: Four times or four $\frac{1}{2}$ IQRs, which would be 2 IQRs.

- ***What is the length of the dotted segments extending from either end of the box?***

  Answer: 34

- ***Move in $\frac{1}{2}$ IQR. Now how far do the dotted segments extend from either end of the box?***

  Answer: 25.5

Student Activity Questions—Activity 1

1. **Select *menu> Class> Set 2* to look at the number of text messages sent/received from students in another class. Select *Summarize*, then $UQ - LQ = IQR$.**

   a. **What is $\frac{1}{2}$ IQR?**

   Answer: 10.75

   b. **Estimate how many times you will have to *Move boundary out 1/2 IQR* before none of the dots will be left outside of the dotted segments.**

   Answers will vary. Students might estimate four or five times.

   c. **Select $UQ - LQ = IQR$. Check your conjecture to b. How long are the dotted segments?**

   Answer: You can only move out five times and one dot is still orange. The total length of the dotted segments on each side of the box is 53.75 messages in length.

2. **Sometimes it is important to determine whether a value is extreme and really distinct from the others. These values are called *outliers*. An outlier is any value that lies outside of the box by a distance of three $\frac{1}{2}$ IQRs in either direction.**

   a. **How many IQRs is three $\frac{1}{2}$ IQRs?**

   Answer: $1.5 \cdot \text{IQR}$ or $1\frac{1}{2} \cdot \text{IQR}$

   b. **Use *Move boundary in 1/2 IQR* and *Move boundary out 1/2 IQR* to identify the students who are outliers in terms of the number of text messages they send/receive in this class.**

   Answer: Two of the students are outliers; the one who sends/receives over 130 messages and the one who sends/receives a bit less than 110.

Student Activity Questions—Activity 1 (continued)

c. **Select *Box Plot/Outliers*. How can you tell from the box plot which students are outliers with respect to the number of texts they send/receive?**

Answer: The box plot looks like a regular plot, but the outliers are not part of the segment. They are represented as dots. Note outliers are still part of the data set and in the upper fourth quarter of the data. The box plot representation has been adapted to make the outliers visible.

d. **Give a plausible explanation for why these students might be outliers.**

Answers will vary. Students might suggest they were involved in organizing something and had to send a lot of messages to people to get the event organized. Another explanation might be that the students are addicted to texting.

3. **Reset. Select *Set 1*. Show the box plot with outliers for the distribution.**

a. **Identify students in this class who are outliers.**

Answer: The student who sends/receives less than 60 messages a day is an outlier.

b. **Give a plausible explanation for why students might be outliers.**

Answer: Perhaps his parents restricted his time texting or he was ill. It could have been an error in recording the data.

c. **Select *Set 3*. Do you think the distribution will have any outliers? Use the TNS activity to check your thinking.**

Answers will vary. None of the students in this class are outliers.

4. **Mathematicians like formulas.**

a. **If *x* represents a value, which of the formulas below do you think can be used to find an outlier? Explain your reasoning.**

i. $x > UQ$ ii. $x > UQ + IQR$

iii. $x > UQ + \frac{1}{2}IQR$ iv. $x > UQ + \frac{3}{2}IQR$

Answer: iv. Not every value beyond the UQ is an outlier; it has to be far away from the other values. The right one is the value that is more than $\frac{3}{2}$ IQRs above the UQ.

b. **Use your reasoning from question a above to write a formula to describe an outlier at the left end of a distribution.**

Answer: $x < \text{LQ} - 1.5 \cdot \text{IQR}$

Student Activity Questions—Activity 1 (continued)

c. **If $LQ=135$, $UQ=185$, identify each as true or false. An outlier will be any value**

i. **larger than 185.**

ii. **larger than 210.**

iii. **larger than 260.**

iv. **smaller than 60.**

Answer: iii and iv are true.

5. **Which of the following are true? Give an example from the TNS activity to support your reasoning.**

a. **The smallest and largest values of any distribution are outliers.**

Answer: That was not true in any of the examples we explored.

b. **Not all distributions have outliers.**

Answer: True, one of the distributions did not have any values beyond three $\frac{1}{2}$ IQRs above or below the box.

c. **An outlier will be more than one box plot width plus half of the width of the box plot to the left and right of the box.**

Answer: True, one box is an IQR and so it is the same as adding an IQR plus $\frac{1}{2}$ IQR, which is $1\frac{1}{2}\cdot IQR$, to the UQ or subtracting it from the LQ.

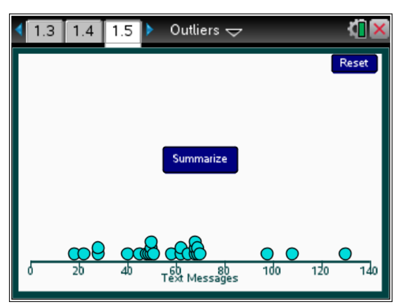
d. **The segments on each side of the box always extend $1\frac{1}{2}\cdot IQR$ beyond the LQ and the UQ.**

Answer: False, the segments go only until the last data point that is not an outlier, which is what they did on all the examples.

Part 2, Page 1.5

Focus: Students will identify outliers both visually and using the formula and noticing the impact of outliers on the mean and mean absolute deviation. Students should recognize that outliers have no impact on the median and interquartile range.

Page 1.5 functions in the same way as page 1.3, except each data set is different.



Class Discussion

Choose* menu> New Class *then select* Summarize *and* UQ – LQ = IQR*.

- ***Identify any outliers in the distribution.***

  Answers will vary. One class had two outliers at the left at 20 and 21 messages.

- ***Record the mean, MAD, and median for your distribution.***

  Answers will vary. In the example above, the mean was 64 messages, the MAD was 14.8 messages, and median about 70 messages.

- ***Suppose the student whose data for median number of messages was at the median really sent 30 more messages that day. Move a point on or near the median to reflect the actual number of messages for that student. Did this change the outliers? The summary measures?***

  Answers will vary. In the example above, the shifted point became an outlier. The mean became 65.1 messages, the MAD was 15.6 messages, and the median did not change.

- ***Suppose a student with 60 text messages really had 55. Which summary measures changed the most? The least?***

  Answers will vary. The changes should not be very large for the mean and MAD, and the median will only change if shifting the 60 crosses a quartile.

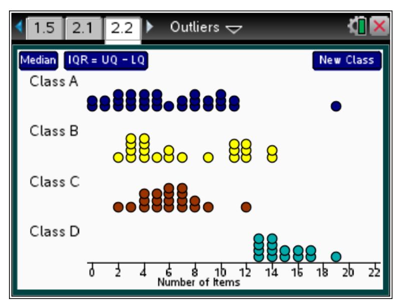
Student Activity Question—Activity 2

1. Work with a partner. Write a short description of the effect of outliers on the measures of center and spread. Use *menu> New Class* to find distributions that support your thinking.

Answers will vary. Students should note that outliers will typically have little effect on the median or the IQR but will influence the mean and MAD.

Part 2, Page 2.2

Focus: Students will identify outliers both visually and using the formula and noticing the impact of outliers on the mean and mean absolute deviation. Students should recognize that outliers have no impact on the median and interquartile range.



On page 2.2, the distributions represent the number of items students in four different classes brought for donations to a food bank.

Median and **IQR = UQ – LQ** show the median/ interquartile range for each distribution of items.

New Class generates distributions of items for a variety of new classes.

Reset returns to the original data.

Class Discussion

The distributions on page 2.2 are the number of items students in four different classes brought for donations to a food bank.

- ***Which of the classes seems to have brought the most items? Make a frequency table and use it to support your answer.***

  Answer: Class A had a lot of students, but all the students in Class D brought at least 13 items.

Class A	Freq	Total Items
0	2	0
1	2	2
2	3	6
3	3	9
4	3	12
5	3	15
6	1	6
7	2	14
8	3	24
9	2	18
10	3	30
11	2	22
19	1	19
Sum		177

Class D	Freq	Total Items
13	4	52
14	4	56
15	2	30
16	2	32
17	2	34
19	1	19
Sum		223

- ***Did any of the classes have a student who really brought a lot or hardly any items compared to the rest of the class? Explain.***

  Answers may vary. One student in Class A brought 19 items, 8 more than the other students in the class. One student in Class C brought 12 items, but that was only three more than the next student.

- ***How could you find an answer to the answer above?***

  Answer: Find out if there is an outlier.

- ***Do you think any of the Classes will have the same IQR? Why or why not? Select*** $\mathbf{IQR = UQ - LQ}$ ***to check your thinking.***

  Answers will vary. Class A and Class B are close (6 and 7 respectively), but Class C at 2.5 and Class D at 3 are even closer to having the same IQR.

Student Activity Questions—Activity 3

1. **Look at the distributions with the IQR segments.**

   a. **Which distributions will probably not have an outlier? Explain how you know.**

   Answers may vary. Students may think Classes B and D seem unlikely to have an outlier because there is no extreme point in either distribution.

   b. **Find 1.5 IQR for each of the classes.**

   Answer: Class A—9; Class B—10.5; Class C—3.75; Class D—4.5

   c. **Use your work from the question above to estimate whether each distribution has an outlier.**

   Answer: An outlier for Class A would be greater than the $\text{UQ} + 1.5 \cdot \text{IQR} = 9 + 9 = 18$, so the value at 19 is an outlier. An outlier for Class C would be greater than 10.75, so 12 items would be an outlier in that class.

2. **Select *New Classes*.**

   a. **Decide if students in any of the classes brought a lot more or a lot less items than their classmates. Use the IQR to help you decide. Check your work with a partner.**

   Answers will vary. In one example, Class C had one student who only brought 2 items (1.5 IQR was 4.5) and Class D had one student who only brought 1 item (1.5 IQR was 6).

   b. **Determine the total number of items brought by one of the classes. Calculate the mean number of items for that class. Explain how you found your answer.**

   Answers will vary. Some students might set up a frequency table. In one set, one class had four students who brought 7 items, two brought 8, one brought 9 and one who brought 19. The total would be $28 + 16 + 9 + 19 = 72$ and to find the mean number of items, $\frac{72}{8} = 9$.

   c. **Select *New Classes* until you find two of the four classes with outliers. Check your work with a partner.**

   Answers will vary.

Deeper Dive — page 2.2

In one class, the IQR was 0. How could that happen?

Answer: All of the students in the middle 50% for items brought, brought exactly the same number of items.

The digits in the test score for one of 25 students were reversed, which made that score an outlier. Including the score with the reversed digits, the maximum score was 98, median 80, upper quartile 88, lower quartile 72 and the minimum score was 46.

- ***Which score was an outlier?***

  Answer: 46

- ***If the correct score was entered, would it still be an outlier?***

  Answer: No because the $1.5(88-72)=24$ and $72-24=48$. 64 is greater than 48, so it would no longer be an outlier.

- ***Which of the five number summary statistics would be affected when the correct score is entered?***

  Answer: Only the minimum, which would no longer be 46

Work with a partner using page 2.2. Each of you should decide whether any of the classes have an outlier by estimating the median and IQR. Then compare your answers and discuss your strategies. Select $\mathbf{IQR = UQ - LQ}$ ***to check your thinking. Choose* New Classes *and try again. Do this several times until you both guess correctly for at least three of the four distributions.***

Answers will vary. Listen to student strategies for ones you would like them to share with the whole class.

Investigate outliers in statistical analyses of data from real contexts and identify at least three reasons for the existence of outliers and give examples from the particular contexts.

Answers will vary. Websites such as http://pareonline.net/getvn.asp?v=9&n=6 or https://savionline.wordpress.com/2013/07/24/the-importance-of-outliers/ can be used to answer the question.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. The table below shows the number of customers at a Bike Shop for 10 days, as well as the mean (average) and the median number of customers for these 5 days.

Day	Customers	
	100	
	95	
	90	
	10	
	80	
	75	
	91	
	110	
	92	
	88	
Summary	Lower quartile = 80 Median = 90.5 Upper quartile = 95	Mean = 83.1 Mean absolute deviation = 16.86

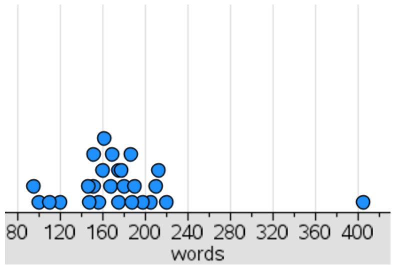
a. Is the 10 an outlier? Explain your reasoning.

Answer: Yes because the IQR = 15 and 1.5(15) = 22.5; 80 – 22.5 = 57.5. 10 < 57.5 so 10 is an outlier.

b. Which summary statistics are most affected by the 10?

Answer: The mean and MAD

2. Researchers surveyed 26 two-year old children to find out the number of words they knew. The results are in the dot plot.



Which of the following might explain the outlier?

a. An older child was interviewed and counted in the summary data.

b. The researchers did not interview enough children.

c. The child probably lied about the words he knew.

d. The child made up a lot of words.

Answer: a. An older child was interviewed and counted in the summary data.

3. To find an outlier you need to know

I. The median

II. The minimum and maximum

III. The quartiles

a. All of the above

b. I and III

c. II and III

d. I and II

Answer: c. II and III

Student Activity Solutions

In these activities you will use outliers to analyze data sets. After completing the activities, discuss and/or present your findings to the rest of the class.

Activity 1 [Page 1.3]

1. Select **menu> Class> Set 2** to look at the number of text messages sent/received from students in another class. Select **Summarize**, then $\text{UQ} - \text{LQ} = \text{IQR}$.

   a. What is $\frac{1}{2}$ IQR?

   Answer: 10.75

   b. Estimate how many times you will have to **Move boundary out 1/2 IQR** before none of the dots will be left outside of the dotted segments.

   Answers will vary. Students might estimate four or five times.

   c. Select $\mathbf{UQ - LQ = IQR}$. Check your conjecture to b. How long are the dotted segments?

   Answer: You can only move out five times and one dot is still orange. The dotted segments are 53.75 messages in length.

2. Sometimes it is important to determine whether a value is extreme and really distinct from the others. These values are called *outliers*. An outlier is any value that lies outside of the box by a distance of three $\frac{1}{2}$ IQRs in either direction.

   a. How many IQRs is three $\frac{1}{2}$ IQRs?

   Answer: $1.5 \cdot \text{IQR}$ *or* $1\frac{1}{2} \cdot \text{IQR}$

   b. Use **Move boundary in 1/2 IQR** and **Move boundary out 1/2 IQR** to identify the students who are outliers in terms of the number of text messages they send/receive in this class.

   Answer: Two of the students are outliers; the one who sends/receives over 130 messages and the one who sends/receives a bit less than 110.

   c. Select **Box Plot/Outliers**. How can you tell from the box plot which students are outliers with respect to the number of texts they send/receive?

   Answer: The box plot looks like a regular plot, but the outliers are not part of the segment. They are represented as dots. Note outliers are still part of the data set and in the upper fourth quarter of the data. The box plot representation has been adapted to make the outliers visible.

   d. Give a plausible explanation for why these students might be outliers.

   Answers will vary. Students might suggest they were involved in organizing something and had to send a lot of messages to people to get the event organized. Another explanation might be that the students are addicted to texting.

3. Reset. Select **Set 1**. Show the box plot with outliers for the distribution.

   a. Identify students in this class who are outliers.

   Answer: The student who sends/receives less than 60 messages a day is an outlier.

   b. Give a plausible explanation for why students might be outliers.

   Answer: Perhaps his parents restricted his time texting or he was ill. It could have been an error in recording the data.

   c. Select **Set 3**. Do you think the distribution will have any outliers? Use the TNS activity to check your thinking.

   Answers will vary. None of the students in this class are outliers.

4. Mathematicians like formulas.

   a. If *x* represents a value, which of the formulas below do you think can be used to find an outlier? Explain your reasoning.

   i. $x > UQ$ ii. $x > UQ + IQR$

   iii. $x > UQ + \frac{1}{2}IQR$ iv. $x > UQ + \frac{3}{2}IQR$

   Answer: iv. Not every value beyond the UQ is an outlier; it has to be far away from the other values. The right one is the value that is more than $\frac{3}{2}$ IQRs above the UQ.

   b. Use your reasoning from question a above to write a formula to describe an outlier at the left end of a distribution.

   Answer: $x < \text{LQ} - 1.5 \cdot \text{IQR}$

   c. If $LQ{=}135$, $UQ{=}185$, identify each as true or false. An outlier will be any value

   i. larger than 185.

   ii. larger than 210.

   iii. larger than 260.

   iv. smaller than 60.

   Answer: iii and iv are true.

5. Which of the following are true? Give an example from the TNS activity to support your reasoning.

   a. The smallest and largest values of any distribution are outliers.

   Answer: That was not true in any of the examples we explored.

   b. Not all distributions have outliers.

   Answer: True, one of the distributions did not have any values beyond three $\frac{1}{2}$ IQRs above or below the box.

c. An outlier will be more than one box plot width plus half of the width of the box plot to the left and right of the box.

Answer: True, one box is an IQR and so it is the same as adding an IQR plus $\frac{1}{2}$ *IQR, which is* $1\frac{1}{2}\cdot IQR$ *, to the UQ or subtracting it from the LQ.*

d. The segments on each side of the box always extend $1\frac{1}{2}\cdot IQR$ beyond the LQ and the UQ.

Answer: False, the segments go only until the last data point that is not an outlier, which is what they did on all the examples.

Activity 2 [Page 1.5]

1. Work with a partner. Write a short description of the effect of outliers on the measures of center and spread. Use **menu> New Class** to find distributions that support your thinking.

Answers will vary. Students should note that outliers will typically have little effect on the median or the IQR but will influence the mean and MAD.

Activity 3 [Page 2.2]

1. Look at the distributions with the IQR segments.

a. Which distributions will probably not have an outlier? Explain how you know.

Answers may vary. Students may think Classes B and D seem unlikely to have an outlier because there is no extreme point in either distribution.

b. Find 1.5 IQR for each of the classes.

Answer: Class A—9; Class B—10.5; Class C—3.75; Class D—4.5

c. Use your work from the question above to estimate whether each distribution has an outlier.

Answer: An outlier for Class A would be greater than the $UQ+1.5\cdot IQR=9+9=18$*, so the value at 19 is an outlier. An outlier for Class C would be greater than 10.75, so 12 items would be an outlier in that class.*

2. Select **New Classes**.

   a. Decide if students in any of the classes brought a lot more or a lot less items than their classmates. Use the IQR to help you decide. Check your work with a partner.

   Answers will vary. In one example, Class C had one student who only brought 2 items (1.5 IQR was 4.5) and Class D had one student who only brought 1 item (1.5 IQR was 6).

   b. Determine the total number of items brought by one of the classes. Calculate the mean number of items for that class. Explain how you found your answer.

   Answers will vary. Some students might set up a frequency table. In one set, one class had four students who brought 7 items, two brought 8, one brought 9 and one who brought 19. The total would be $28+16+9+19=72$ *and to find the mean number of items,* $\frac{72}{8}=9$.

   c. Select **New Classes** until you find two of the four classes with outliers. Check your work with a partner.

   Answers will vary.