

Bonds and Bond Yield to Maturity

A **bond** is a long-term promissory note issued by a corporation. The associated contract, called the bond indenture, specifies that the creditor will receive regular interest payments for the term of the agreement and then receive the **face amount** of the bond. The **coupon rate** is the interest rate.

Objectives:

- Determine the market value of a bond.
- Determine the present value of a bond.
- Explore the impact of interest rates on bond values.
- Determine the net present value of a bond.

Example 1:

A ten-year \$1,000 bond promises 6% per year with annual payments of \$60 per year ($0.06 * \$1,000$) for 10 years. On the maturity date, the lender will be paid the maturity value of \$1,000. The total amount that the lender receives is the original \$1,000 plus \$600 (10 payments of \$60 each). Find the market value of the bond.

The **market value** of a bond is the **present value** (discounted value) of the expected payments (interest and principal).

$$PV = \frac{60}{1.06} + \frac{60}{(1.06)^2} + \dots + \frac{60}{(1.06)^{10}} + \frac{1000}{(1.06)^{10}} = 441.61 + 558.39 = 1000$$

In the above expression, the first ten terms represent the present value of the interest payments, \$441.61. The last term ($1000/1.06^{10}$ or 558.29) is the discounted value of the \$1,000 lump sum payment. The **TVM Solver** will be used twice to compute the market value of the bond.

1. Press , and select **New** to start a new document. Select **Add Calculator**.

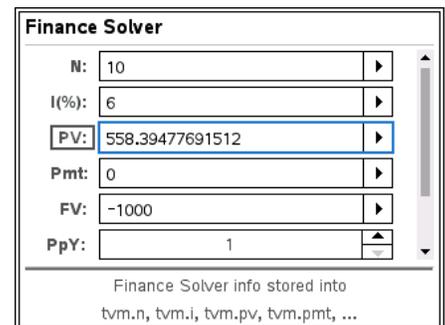
Note: To round computations to two decimal places, change the Display Digits setting in the Documents Settings to **Fix 2**.

2. Press **Menu > Finance**. Select **Finance Solver**.

To move from row to row in the Finance Solver, press .

3. Because compounding occurs once a year, enter $N = 10$, $I(\%) = 6$, $Pmt = 0$, and $FV = -1000$.
4. Calculate the present value by placing the cursor in the Present Value (PV) row and pressing .

Arrow to another row to display the present value rounded to two decimal places.



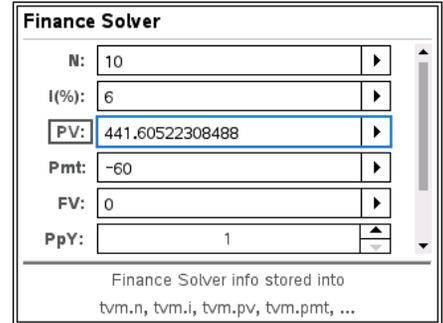
The present value of the \$1,000 lump sum payment is \$558.39.

5. For the present value of the 10 payments of \$60 each year, use the **Finance Solver**, and input $FV = 0$ and $Pmt = -60$.
6. Calculate the Present Value by placing the cursor in the Present Value (PV) row and pressing **enter**.

Notice the sum of \$441.61 and \$558.39 equals the market value of the \$1,000 bond.

This calculation could have been done in one step with the **Finance Solver**.

7. Set $Pmt = -60$ and $FV = -1000$. The other values are the same as those given above.
8. Place the cursor in the Present Value (PV) row and press **enter**.



Finance Solver

N: 10

I(%): 6

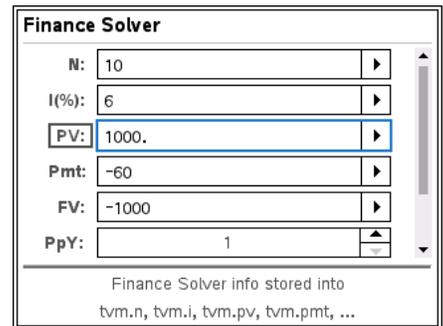
PV: 441.60522308488

Pmt: -60

FV: 0

PpY: 1

Finance Solver info stored into
tvm.n, tvm.i, tvm.pv, tvm.pmt, ...



Finance Solver

N: 10

I(%): 6

PV: 1000.

Pmt: -60

FV: -1000

PpY: 1

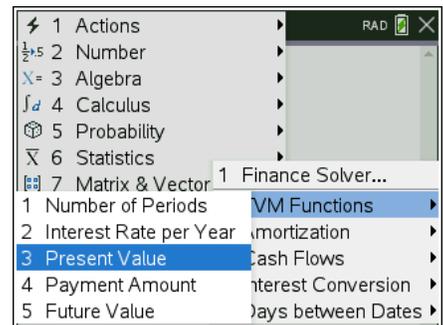
Finance Solver info stored into
tvm.n, tvm.i, tvm.pv, tvm.pmt, ...

Another method for calculating the present value is to use the **tvmPV** command. The syntax for this function is **tvmPV(N, I(%), Pmt, FV, PpY, CpY)**.

9. Press **esc** to exit the Finance Solver.
10. On the Calculator page, press **Menu > Finance**, and select **TVM Functions**.
11. Select **Present Value**.

This will paste the **tvmPV** function on the Calculator page.

12. Enter **10,6,-60,-1000,1,1)**. Press **enter**.



1 Actions

2 Number

3 Algebra

4 Calculus

5 Probability

6 Statistics

7 Matrix & Vector

1 Finance Solver...

1 Number of Periods

2 Interest Rate per Year

3 Present Value

4 Payment Amount

5 Future Value

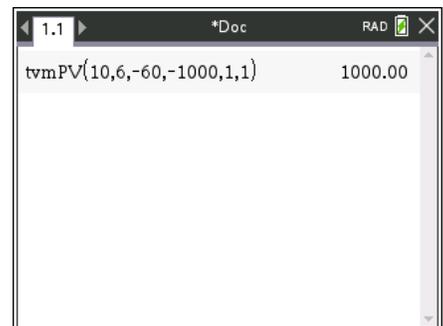
1 VM Functions

2 amortization

3 Cash Flows

4 Interest Conversion

5 Days between Dates



1.1

*Doc

tvmPV(10,6,-60,-1000,1,1) 1000.00



Exploration of Bond Value Versus Current Market Interest Rates

If the current market interest rate is the same as the coupon rate on bonds of similar quality, then the market value of the bond equals the face amount of the bond. If current market interest rates are higher, the market value of the bond is lower, and the bond is selling at a discount. If current market interest rates are lower, the market value of the bond is higher, and the bond is selling at a premium.

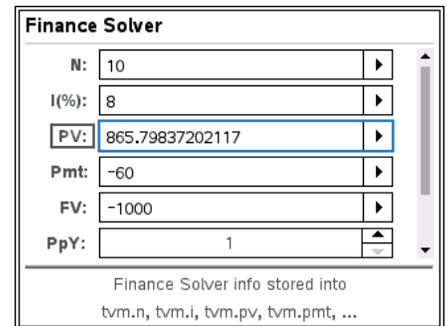
Example 2:

Find the present value of the \$1,000 bond in Example 1 if the current market interest rate is 8%.

Use the **Finance Solver**.

1. Press **Menu > Finance**. Select **Finance Solver**.
2. Enter $N = 10$, $I(\%) = 8$, $Pmt = -60$, $FV = -1000$, $PpY = 1$, and $CpY = 1$.
3. Place the cursor in the Present Value (PV) row and press **enter**.

The present value of the bond is \$865.80.



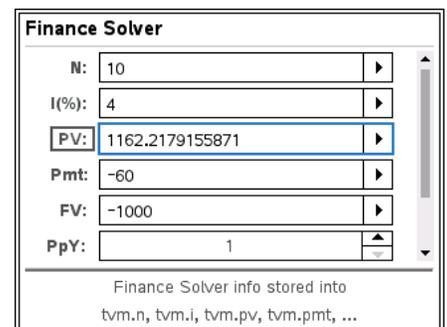
Example 3:

If the current market rate for similar bonds is 4%, what should an investor expect to pay for a \$1,000, 6% 10-year bond?

Use the **Finance Solver**.

1. Press **Menu > Finance**. Select **Finance Solver**.
2. Enter $N = 10$, $I(\%) = 4$, $Pmt = -60$, $FV = -1000$, $PpY = 1$, and $CpY = 1$.
3. Place the cursor in the Present Value (PV) row and press **enter**.

The investor should expect to pay \$1,162.22.



Press **esc** to exit the Finance Solver.



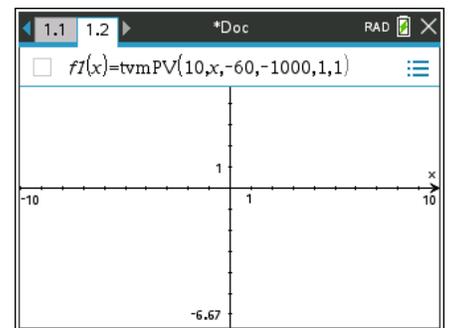
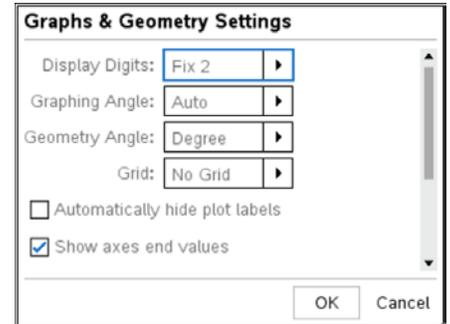
To visualize the impact of interest rate on bond values, explore the graph of the present value of the bond as a function of interest rate.

Let $f_1(x)$ be the sum of the present value of the interest payments and the present value of the \$1,000 repayment of principal.

$f_1(x) = \text{tvmPV}(10, x, -60, -1000, 1, 1)$ where x is the interest rate.

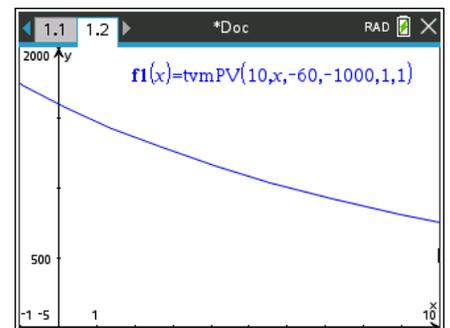
4. Press **ctrl** **doc** to add a new page. Select **Add Graphs**.
5. Press **Menu > Settings**, and change Display Digits to **Fix 2**. Select **OK** or press **enter**.
6. In $f_1(x)$, enter **tvmPV(10,x,-60,-1000,1,1)**, and press **enter**.

Note: Select **tvmPV** from the catalog.



7. Press **Menu > Window/Zoom > Window Settings**. Enter XMin = -1, XMax = 10, XScale = 1, YMin = -5, YMax = 2000, and YScale = 500. Select **OK** or press **enter**.

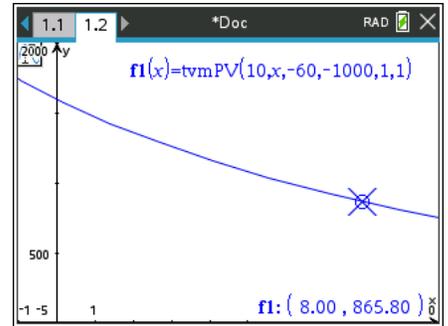
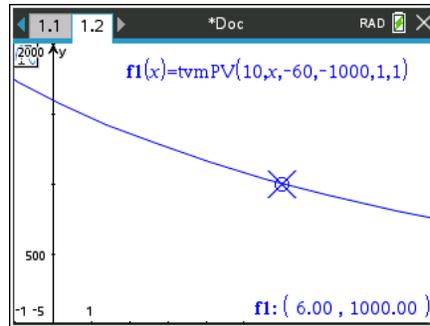
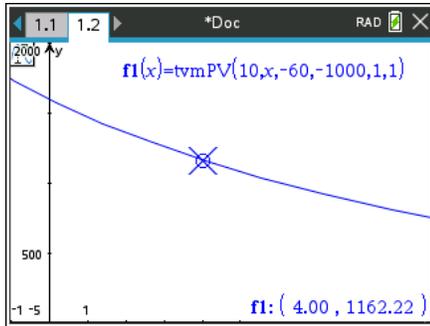
Note: The function label was moved to better display the graph.



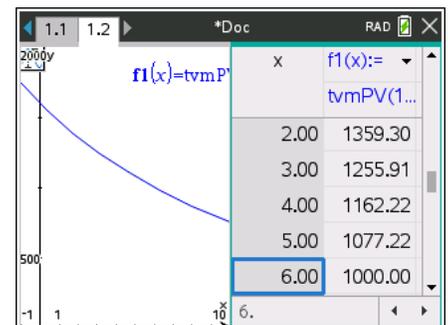
8. Press **Menu > Trace > Graph Trace**. Use the arrow keys to move the cursor along the curve.

Type in a specific x value (for example, 4), and press **enter**.

Explore other values for x, the interest rate.



9. To see a table of interest rates and present values, press **Menu > Table > Split-screen Table**.
10. Press **Menu > Table > Edit Table Settings**. Enter 1 for the Table Start value and 1 for the Table Step.
11. Select **OK** or press **enter** to display a table of the present value of the bond at various interest rates.



Example 4:

XYZ Corporation issued a 30-year \$1,000 bond in 2014 with a coupon rate of 3.9%. What was it worth in 2022 if the current interest rate for similar investments in 2022 was 4.92%?

1. Press , and select **New** to start a new document. Select **Add Calculator**.

Note: To round computations to two decimal places, change the Display Digits setting in the Documents Settings to **Fix 2**.

2. Press **Menu > Finance > Cash Flows**. Select **Net Present Value**.

The syntax for this command is **npv**(interest rate, initial cash flow, cash flow list, frequency of cash flow entries).

In this example, the cash flow list is \$39.00 per year; the last cash flow is principal and interest, or \$1,039.00. The frequency list is {21,1} because \$39.00 is issued 21 times and \$1,039.00 is used once.

3. Enter **4.92, 0, {39, 1039}, {21,1}**. Press .

The net present value in 2022 was \$864.75.

