

## Breakeven Analysis

Breakeven analysis is concerned with the relationship between the cost and revenue of an enterprise. Implicit in such an analysis is an examination of the relationship between fixed and variable costs, profits, and pricing policy and the volume of output (quantity produced).

Such an analysis of prices, costs, and profits is good for the short term (usually twelve months). The analysis is only as good as the model that is built; the more flexible the pricing policy, the more difficult the model.

### Objectives:

- Determine the quantity of a product that must be sold to break even.

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The following quantities are important in the breakeven analysis:

- F Fixed Cost
- V Variable cost per unit
- P Unit Price
- X Number of units
- T Pre-tax profit

Revenue  $R(X)$  and costs  $C(X)$  are both functions of  $X$ , the number of units.

$$\text{Revenue} = R(X) = P * X$$

$$\text{Cost} = C(X) = F + V * X$$

$$\text{Profit} = T(X) = R(X) - C(X) = P * X - (F + V * X)$$

If  $R(X)$  and  $C(X)$  are each linear relationships, with  $R(X) = P * X$  and  $C(X) = F + V * X$ , then

$$\text{Profit} = P * X - F - V * X = (P - V)X - F.$$

The above model is linear based on the quantity  $X$ . Breakeven analysis determines the quantity  $X$  for which the pre-tax profit is zero. Hence, the problem becomes

$$\text{Pre-tax profit} = (P - V)X - F = 0$$

where we must solve for  $X$ , given values for  $P$  (price per unit),  $V$  (variable cost per unit), and  $F$  (fixed cost).

On the TI-Nspire™ CX, the breakeven problem can be solved for the number of units  $X$  in several ways:

- numerically, using the Numerical Solve command
- graphically
- numerically, with tables and inspection

**Example 1:**

Rugged Can Company sells trash cans for \$20 each. Each unit has a variable cost of \$15 and the fixed costs are \$4,000. How many cans must be sold to break even?

Method 1: Numerical Solve

Use  $(P - V)X - F = 0$  with  $P = 20$ ,  $V = 15$  and  $F = 4000$ .

1. Press , and select **New** to start a new document. Select **Add Calculator**.

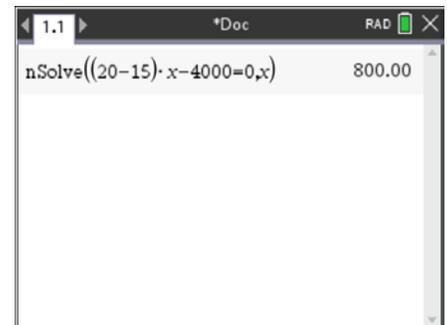
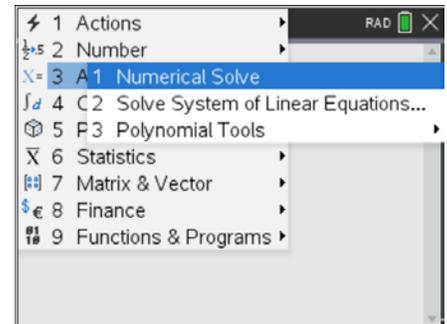
**Note:** To round computations to two decimal places, change the Display Digits setting in the Documents Settings to **Fix 2**.

2. Press **Menu > Algebra**. Select **Numerical Solve**.

The syntax is **nSolve**(Equation, Var[=Guess]). A guess is optional.

3. Enter  $( ( 20 - 15 ) \times X - 4000 = 0 )$  and press .

Breakeven occurs if 800 units are produced and sold.



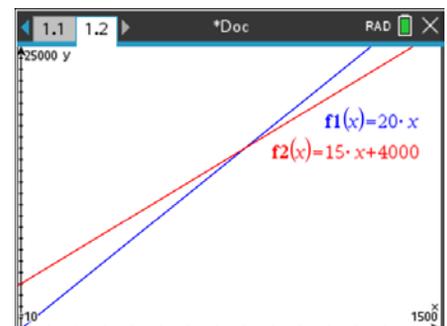
Method 2: Graphing

1. Add a Graphs page to a current document by pressing   and selecting **Add Graphs**.

The Revenue function can be entered in  $f1(x)$  and the Cost function in  $f2(x)$ , either by using the actual numbers ( $f1(x) = 20 \cdot X$  and  $f2(x) = 15 \cdot X + 4000$ ) or, in general, by entering  $f1(x) = P \cdot X$  and  $f2(x) = V \cdot X + F$ .

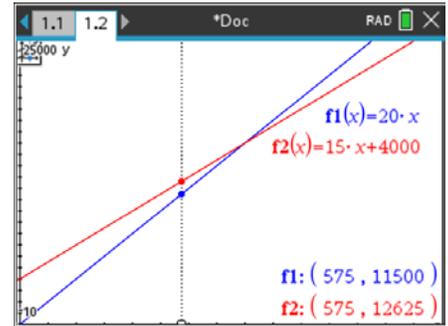
**Note:** If the functions are entered in terms of  $P$ ,  $V$ , and  $F$ , values for  $P$ ,  $V$ , and  $F$  must be first stored on a Calculator page, e.g.,  $20$    **P** .

2. Enter  $f1(x) = 20 \cdot X$ , press , and enter  $f2(x) = 15 \cdot X + 4000$ . Press .
3. Choose an appropriate viewing window for these values. Press **Menu > Window/Zoom > Window Settings**. Enter  $XMin = -10$ ,  $XMax = 1500$ ,  $XScale = 100$ ,  $YMin = -10$ ,  $YMax = 25000$ , and  $YScale = 1000$ . Select **OK** or press .



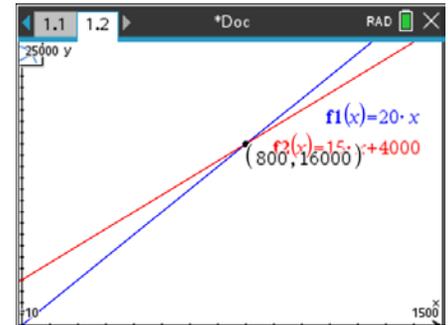
- Press **Menu > Settings**, and change the Display Digits to **Float**.
- Press **Menu > Trace > Trace All** to see the values of X and the function values evaluated at X. The left and right arrows move the cursor along the functions. Notice that the function values are given at the bottom of the screen on the right.

**Note:** To show the function value at a particular value of X, type the X value, and press **enter**.



- To find the point of intersection, press **Menu > Geometry > Points & Lines > Intersection Point(s)**. Click on the first graph and then the second graph.

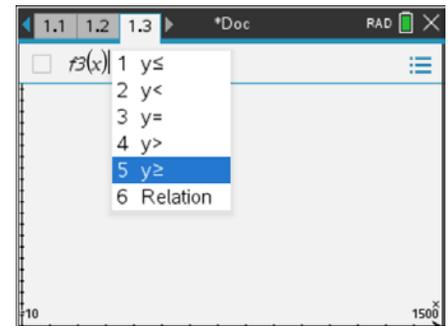
The coordinates of the intersection point, (800, 16000), are displayed, indicating a breakeven point at X = 800.



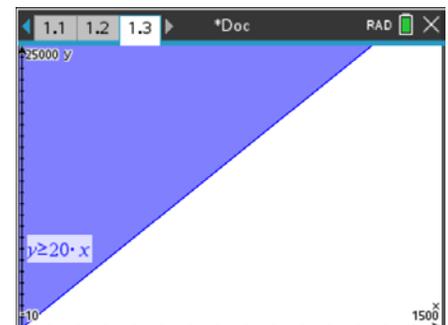
- To graphically show the loss, add a Graphs page by pressing **ctrl doc** and selecting **Add Graphs**.

- Press **Menu > Window/Zoom > Window Settings**. Enter XMin = -10, XMax = 1500, XScale = 100, YMin = -10, YMax = 25000, and YScale = 1000. Select **OK** or press **enter**.

- Press **del** to delete the equals sign. Arrow to  $y \geq$  and press **enter**.



- Enter 20x and press **enter**.



11. Press **[tab]**, and enter  $y \leq 15x + 4000$ . Press **[enter]**.

**Note:** Press **[ctrl]** **[=]** to access the  $\leq$  symbol.

To display the point of intersection, press **Menu > Trace > Trace All**, enter 800, and press **[enter]**.

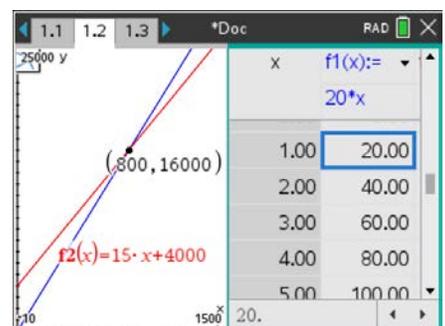
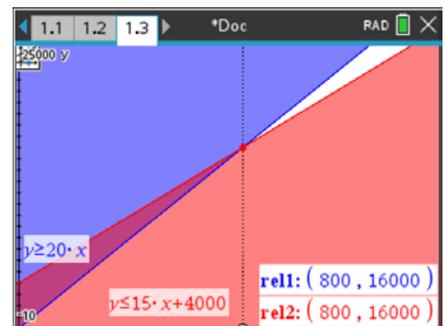
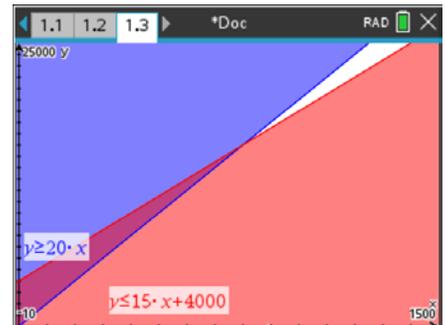
The area with both types of shading represents the net loss (negative profit). This region, to the lower left of the breakeven point, shows the difference between the Cost line and the Revenue line.

Conversely, the area with no shading to the upper right of the breakeven point represents the net profit. This area, between the two lines, shows the difference between the Revenue line and the Cost line, and represents the profit area.

### Method 3: Tables

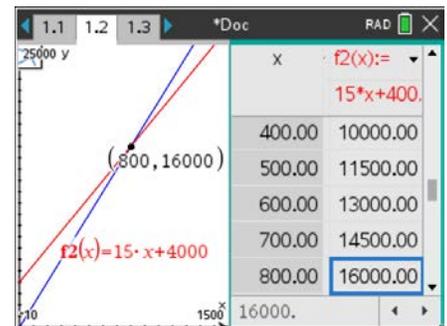
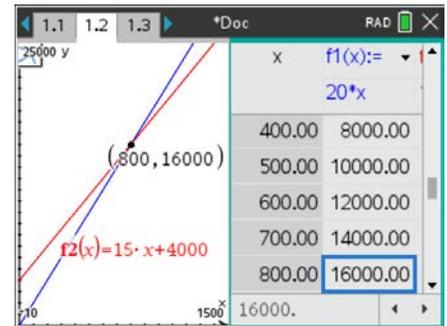
Once  $f_1(x)$  and  $f_2(x)$  have been defined as Revenue and Cost, respectively, tables can give a numerical visualization of the problem.

1. Press **[ctrl]** **[↔]** or click on tab 1.2 to move to page 1.2.
2. Press **[ctrl]** **[T]** to display a function table. Alternatively, press **Menu > Table**, and select **Split-screen Table**.
3. Press **Menu > Table**, and select **Edit Table Settings**. Let Table Start be 0 and Table Step be 100. X, the number of items, can never be negative. The choice of 100 for the table increment, Table Step, is convenient for this problem.

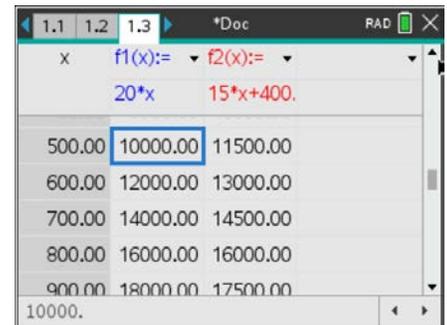


4. Select **OK**.
5. Arrow to the right to display function values for  $f_2(x)$ .

Notice that at 800, the values in  $f_1(x)$  and  $f_2(x)$  are both 16,000. Breakeven occurs when revenue equals cost.



**Note:** To display the table on a separate page, press **doc** and select **Page Layout > Ungroup**. Move to the next page (page 1.3) to view the table.



If either or both of the cost and revenue functions are non-linear, these same methods will apply.