

TI in Focus: AP[®] Calculus

2017 AP[®] Calculus Exam: AB-2
Scoring Guidelines

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Outline

- (1) Free Response Question
- (2) Scoring Guidelines
- (3) Student performance
- (4) Interpretation
- (5) Common errors
- (6) Specific scoring examples

2. When a certain grocery store opens, it has 50 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

$$f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right) \text{ for } 0 < t \leq 12,$$

where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

$$g(t) = 3 + 2.4 \ln(t^2 + 2t) \text{ for } 3 < t \leq 12,$$

where $g(t)$ is measured in pounds per hour and t is the number of hours after the store opened.

- (a) How many pounds of bananas are removed from the display table during the first 2 hours the store is open?
- (b) Find $f'(7)$. Using correct units, explain the meaning of $f'(7)$ in the context of the problem.
- (c) Is the number of pounds of bananas on the display table increasing or decreasing at time $t = 5$? Give a reason for your answer.
- (d) How many pounds of bananas are on the display table at time $t = 8$?

$$(a) \int_0^2 f(t) dt = 20.051175$$

20.051 pounds of bananas are removed from the display table during the first 2 hours the store is open.

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

$$(b) f'(7) = -8.120 \text{ (or } -8.119 \text{)}$$

After the store has been open 7 hours, the rate at which bananas are being removed from the display table is decreasing by 8.120 (or 8.119) pounds per hour per hour.

$$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{meaning} \end{cases}$$

$$(c) g(5) - f(5) = -2.263103 < 0$$

Since $g(5) - f(5) < 0$, the number of pounds of bananas on the display table is decreasing at time $t = 5$.

$$2 : \begin{cases} 1 : \text{considers } f(5) \text{ and } g(5) \\ 1 : \text{answer with reason} \end{cases}$$

$$(d) 50 + \int_3^8 g(t) dt - \int_0^8 f(t) dt = 23.347396$$

23.347 pounds of bananas are on the display table at time $t = 8$.

$$3 : \begin{cases} 2 : \text{integrals} \\ 1 : \text{answer} \end{cases}$$

Student Performance

- (1) In general, made the connection between definite integral and net change.
- (2) Misinterpretation of the function $f(t)$.
- (3) Confusion with particle motion problems: use of *velocity*, *acceleration*.
- (4) Imprecise and inaccurate language in explanation.
- (5) Use of derivatives or integrals when not necessary.

Notes

- (1) Calculator active problem:
often we look for the correct set-up, and then the correct answer.
- (2) Set-ups must be given using correct mathematical language and notation.
- (3) Decimal presentation rules.

Rates, Degrees, Units, and dt

- (1) The given functions are rates.
In each part, we must be convinced the student is working with the appropriate function.
(a) and (d): integral, (b) derivative, (c) original.
- (2) Degree mode: eligibility rules.
- (3) Units are required only in part (b).
- (4) Can use expressions for $f(t)$ and $g(t)$.
- (5) A missing dt is assumed to appear to the right of the last term after each integral symbol and before any comparison ($=, \leq, \geq, <, >$).

Part (a) 1: integral

- (1) Conceptual point, correct set-up.
- (2) Ignore all units (correct or incorrect)
- (3) Earned for the correct definite integral.
- (4) Errors in student's expression for $f(t)$ come off the answer point.
- (5) Earned even if extra terms or constants.

$$50 + \int_0^2 f(t) dt = C \qquad 1 - 0$$

$$\int_0^2 [f(t) + 50] dt = C \qquad 0 - 0$$

Part (a) 1: answer

(1) Earned for the correct numerical answer: 20.051

(2) Indefinite integral or no integral and correct answer.

$$\int f(t) dt = 20.051 \qquad 0 - 1$$

(3) Incorrect limits and correct answer (presentation error).

$$\int_0^1 f(t) dt = 20.051 \qquad 0 - 1$$

Part (b) 1: value

- (1) Earned for the value -8.120 . (May be unlabeled)
- (2) This point is in the bank.
- (3) Units are connected to the second point.
- (4) If calculation is done by hand, need not be simplified:

$$0.8 \cdot \sin\left(\frac{7^3}{100}\right) + (0.8 \cdot 7) \cdot \frac{3 \cdot 7^2}{100} \cdot \cos\left(\frac{7^3}{100}\right)$$

Part (b) 1: meaning

- (1) Earned for explaining the meaning in the context of the problem.
- (2) Eligibility: must have produced a value for $f'(7)$.

If the value is incorrect, it must be labeled.

- (3) Must see three items.
 - Correct units of lbs/h².
 - Appeal to the time $t = 7$ (no units on time OK).
 - Evidence of correct interpretation as a rate of a rate in this context.

Part (b) 2nd Point Subtleties

First Condition: $\frac{\text{lbs}}{h^2}$

(1) bananas \neq lbs

(2) Correct: $\frac{p}{h^2}$, $\frac{\text{lbs of bananas}}{hr^2}$

(3) Incorrect: $\frac{\text{bananas}}{h^2}$

Second Condition: $t = 7$

(1) Correct: "... when $t = 7$ ", "... at $t = 7$ "

(2) Incorrect: "... in 7 hours", "... after 7 hours"

Part (b) Other Issues

- (1) Problematic interpretations. (f is changing.)
- Correct: The number of lbs of bananas being removed **per hour** is decreasing (implies $f' < 0$)
 - Incorrect: The number of lbs of bananas being removed is decreasing (implies $f < 0$)
- (2) The context is bananas. Here are some incorrect statements.
- The slope of the rate is negative.
 - The acceleration of bananas removed.
 - The acceleration of customers removing bananas.
- (3) Incorrect Signs.
- The rate is decreasing at a rate of -8.12 lbs/hr². (Incorrect)
 - The rate is changing at a rate of 8.12 lbs/h². (Incorrect)

Part (c) 1: considers $f(5)$ and $g(5)$

- (1) $f(5)$ and $g(5)$ sufficient (no values necessary).
- (2) Verbal descriptions without explicitly stating f and g are too vague.
- (3) Can earn the point if convinced the student is working with $t = 5$ if the correct numerical values are given for $f(5)$, $g(5)$, or $g(5) - f(5)$.
- (4) Must appeal to $f(5)$ and $g(5)$ only.

Integral or derivative of these functions scores 0 - 0.

Part (c) 1: answer with reason

(1) Eligibility:

- Must consider $f(5)$ and $g(5)$.
- If values of $f(5)$ and $g(5)$ are presented, they must be correct.

(0 - 1 not possible)

(2) Minimal answers: (Score 1 - 1)

- $g(5) < f(5)$ decreasing
- $g < f$ at $t = 5$, decreasing
- $g - f = -2.26$, decreasing
- $f(5) = 13.795$, $g(5) = 11.532$, decreasing

Part (d) 2: integrals

- (1) One point for each of the definite integrals: $\int_0^8 f(t) dt$ and $\int_3^8 g(t) dt$
- (2) Attempts to combine the definite integrals and/or incorporate the initial condition are part of the third point.
- (3) If upon applying basic properties of integrals, the resulting expression contains one or both of our definite integrals, award appropriate credit.

$$\bullet \int_0^8 50 + g(t) - f(t) \quad 1/2$$

$$\bullet \int_3^8 g(t) dt - \int_0^8 f(t) dt - \int_0^3 f(t) dt \quad 1/2$$

$$\bullet \int_3^8 [g(t) - f(t)] dt - \int_0^3 f(t) dt \quad 2/2$$

Part (d) 1: answer

(1) Earned for the value 23.347

(2) One exception: neglects the restricted domain of $g(t)$.

$$50 + \int_0^8 [g(t) - f(t)] dt = 41.843 \quad 1 - 1$$

(3) Presentation error: Suppose $k \in (0, 8)$

$$\int_0^k g(t) dt - \int_3^8 f(t) dt + 50 = 23.347 \quad 1 - 1$$

(4) Linkage error:

$$\begin{aligned} 50 + \int_3^8 g(t) dt - \int_0^8 f(t) dt &= 50 + 58.96406 \\ &= 108.96406 - 85.61666 \\ &= 23.347 \quad 2 - 0 \end{aligned}$$

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