

TI in Focus: AP[®] Calculus

2017 AP[®] Calculus Exam: AB-4/BC-4
Scoring Guidelines

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Outline

- (1) Free Response Question
- (2) Scoring Guidelines
- (3) Student performance
- (4) Interpretation
- (5) Common errors
- (6) Specific scoring examples

4. At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius ($^{\circ}\text{C}$) at time $t = 0$, and the internal temperature of the potato is greater than 27°C for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.
- (a) Write an equation for the line tangent to the graph of H at $t = 0$. Use this equation to approximate the internal temperature of the potato at time $t = 3$.
- (b) Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time $t = 3$.
- (c) For $t < 10$, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\frac{dG}{dt} = -(G - 27)^{2/3}$, where $G(t)$ is measured in degrees Celsius and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the potato at time $t = 3$?

$$(a) H'(0) = -\frac{1}{4}(91 - 27) = -16$$

$$H(0) = 91$$

An equation for the tangent line is $y = 91 - 16t$.

The internal temperature of the potato at time $t = 3$ minutes is approximately $91 - 16 \cdot 3 = 43$ degrees Celsius.

3 : $\begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line} \\ 1 : \text{approximation} \end{cases}$

$$(b) \frac{d^2H}{dt^2} = -\frac{1}{4} \frac{dH}{dt} = \left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)(H - 27) = \frac{1}{16}(H - 27)$$

$$H > 27 \text{ for } t > 0 \Rightarrow \frac{d^2H}{dt^2} = \frac{1}{16}(H - 27) > 0 \text{ for } t > 0$$

Therefore, the graph of H is concave up for $t > 0$. Thus, the answer in part (a) is an underestimate.

1 : underestimate with reason

$$(c) \frac{dG}{(G-27)^{2/3}} = -dt$$

$$\int \frac{dG}{(G-27)^{2/3}} = \int (-1) dt$$

$$3(G-27)^{1/3} = -t + C$$

$$3(91-27)^{1/3} = 0 + C \Rightarrow C = 12$$

$$3(G-27)^{1/3} = 12 - t$$

$$G(t) = 27 + \left(\frac{12-t}{3}\right)^3 \text{ for } 0 \leq t < 10$$

The internal temperature of the potato at time $t = 3$ minutes is

$$27 + \left(\frac{12-3}{3}\right)^3 = 54 \text{ degrees Celsius.}$$

$$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration and} \\ \quad \text{uses initial condition} \\ 1 : \text{equation involving } G \text{ and } t \\ 1 : G(t) \text{ and } G(3) \end{cases}$$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

Student Performance

- (1) Contextual, wordy, *internal temperature* appeared 7 times.
- (2) A struggle to understand and incorporate all information.
- (3) Use the DE in each part, and the calculus necessary.
- (4) Variables: x and y versus t and H
- (5) Part (a): confusion around the independent versus the dependent variable.
- (6) Part (b): many students did not use the Chain Rule; local arguments.
- (7) Part (c): separation errors; no constant of integration.

Part (a) 1: slope

- (1) No work needed to earn the point.
- (2) Once earned, in the bank; no label or mislabel: OK

$$-\frac{1}{4}(91 - 27) \quad -16 \quad \frac{dy}{dx} = -16 \quad m = -16 \text{ (change)}$$

- (3) -16 can appear as part of a correct tangent line and earn the first two points.

$$y = -16t + 91 \quad 1 - 1 - ?$$

$$y - 91 = -16(x - 0) \quad 1 - 1 - ?$$

$$H = 91 - 16x \quad 1 - 1 - ?$$

Part (a) 1: tangent line

slope $\neq -16$ OK IF

(1) The line passes through (0, 91) AND

(2) slope ($\neq -16$) = $\frac{dH}{dt}$ or $\frac{dy}{dx}$ (but not m or slope).

Examples

(1) $y = -16t + 91$ 1 - 1 - ?

(2) $y - 91 = -16(x - 0)$ 1 - 1 - ?

(3) $\frac{dy}{dx} = \frac{27}{4}$ $y = \frac{27}{4}x + 91$ 0 - 1 - 0

(4) $\frac{dH}{dt} = -14$ $y = -14t + 91$ 0 - 1 - 0

(5) $m = 16$ $y = 16x + 91$ 0 - 0 - 0

Part (a) 1: approximation

The symbol \approx is not needed.

Examples

$$(1) \frac{dH}{dt} = -16 \quad y = -16t + 91 \quad y = 43 \quad 1 - 1 - 1$$

$$(2) y = -16t + 91 \quad y = 43 \quad 1 - 1 - 1$$

$$(3) H'(0) = -16 \quad H(3) = -16(3) + 91 = 43 \quad 1 - 0 - 1$$

$$(4) \frac{dH}{dt} = -0.25(91 - 27) = -15 \quad y = -15x + 91 \quad y = 46 \quad 1 - 1 - 0$$

$$(5) \frac{dH}{dt} = -0.25(0 - 27) = \frac{27}{4} \quad y = \frac{27}{4}x + 91 \quad y = 111.25 \quad 0 - 1 - 0$$

Part (b) 1: underestimate with reason

(1) Need two items:

(a) The derivative of $\frac{dH}{dt}$.

$$\frac{1}{16}(H - 27); \quad \frac{1}{16}H - \frac{27}{16}; \quad \left(-\frac{1}{4}\right)\left(-\frac{1}{4}\right)(H - 27); \quad -\frac{1}{4}\frac{dH}{dt}$$

(b) The second derivative is positive.

(2) Do not need to appeal to an interval, nor discuss concavity.

(3) Common errors:

Overestimate; $\frac{d^2H}{dt^2} = -\frac{1}{4}$; local argument.

Part (c) 1: separation of variables

Examples

$$(1) -(G - 27)^{-2/3} dG = dt \quad 1 - ? - ? - ? - ?$$

$$(2) \int \sqrt[3]{(G - 27)^{-2}} dG = \int -dt \quad 1 - ? - ? - ? - ?$$

$$(3) \frac{-1}{(G - 27)^{2/3}} = 1 \quad ? - ? - ? - ? - ?$$

$$(4) (G - 27)^{-2/3} \frac{dG}{dt} = -1 \quad 1 - ? - ? - ? - ?$$

$$(5) 3(G - 27)^{1/3} = -t \quad 1 - 1 - ? - ? - ?$$

$$(6) \frac{dG}{dt} = (G - 27)^{2/3} dG \implies G(t) = \frac{3}{5}(G - 27)^{5/3} \quad \text{no separation: } 0/5$$

Part (c) 1: antiderivatives

- (1) One point for two correct antiderivatives: $-3(G - 27)^{1/3} = t$
- (2) OR for two consistent antiderivatives after an honorable attempt at separation.
- (3) Honorable attempts are of this form:
- $$\pm(G - 27)^k dG = \pm dt \quad \text{where } k = \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{3}{2}$$
- (4) Note: Readers have to check the antiderivatives.

Part (c) 1: constant of integration and uses initial condition

- (1) Eligibility: at least one antiderivative correct or consistent.
- (2) $+ C$ must be explicit and $(0, 91)$ must be reasonably clear.
- (3) Note: No constant of integration, max 2/5.

Part (c) 1: equation involving G and t

- (1) Could be implicit or explicit;
must be correct or consistent with antiderivatives.
- (2) Implicit equation must be of the form: $A(G - 27)^B = \pm t + C$
where A is rational, B is a non-integer rational, and C is the numerical value imported from the 3rd point.
- (3) Explicit equation must be of the form: $G = 27 + \left(\frac{C \pm t}{A}\right)^B$
where A is rational, B is rational, and C is the numerical value imported from the 3rd point.

Part (c) 1: $G(t)$ and $G(3)$

- (1) $G(t)$ explicit AND correct or consistent.
- (2) 54 or the consistent $G(3)$ based on their $G(t)$; but $27 < G(3) < 91$.

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