2017 AP Calculus Exam: AB-5 Rectilinear Motion

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Background

(1) Particle motion problems occur often on the AP Calculus Exam.

(2) Applications involve: Fundamental Theorem of Calculus, displacement, total distance traveled, many are calculator active.

Suppose a particle moves along a horizontal line so that its position at time t, $t \geq 0$ is given by $s(t)$.

Velocity: $v(t) = s'(t)$

Acceleration: $a(t) = v'(t) = s''(t)$

Expressions and Mathematical Translations

The particle is at rest; moving to the right; moving to the left.	$v(t) = 0$; $v(t) > 0$; $v(t) < 0$
The average velocity of the particle on the interval $[a, b]$.	$\dfrac{1}{b-a}\displaystyle\int_a^b v(t)\,dt = \dfrac{s(b)-s(a)}{b-a}$
The velocity of the particle is increasing; decreasing.	$a(t) = v'(t) > 0$; $a(t) = v'(t) < 0$
The speed of the particle.	$\lvert v(t)\rvert$
The total distance traveled by the particle over the interval $[a, b]$.	$\displaystyle\int_a^b \lvert v(t)\rvert\,dt$
The displacement of the particle, or net distance traveled by the particle, over the interval $[a, b]$.	$\displaystyle\int_a^b v(t)\,dt$

Example

A particle moves along a horizontal line. For $0 \leq t \leq 8$, the velocity of the particle is given by $v(t) = t - 4e^{-t^2} + 3t\cos\left(\frac{t^2}{12}\right)$, and the position is given by $s(t)$. It is known that $s(0) = -5$.

(a) Find all values of t in the interval $6 \leq t \leq 8$ for which the speed of the particle is 10.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 4$.

(c) Find all times t in the interval $0 \leq t \leq 8$ at which the particle changes direction.

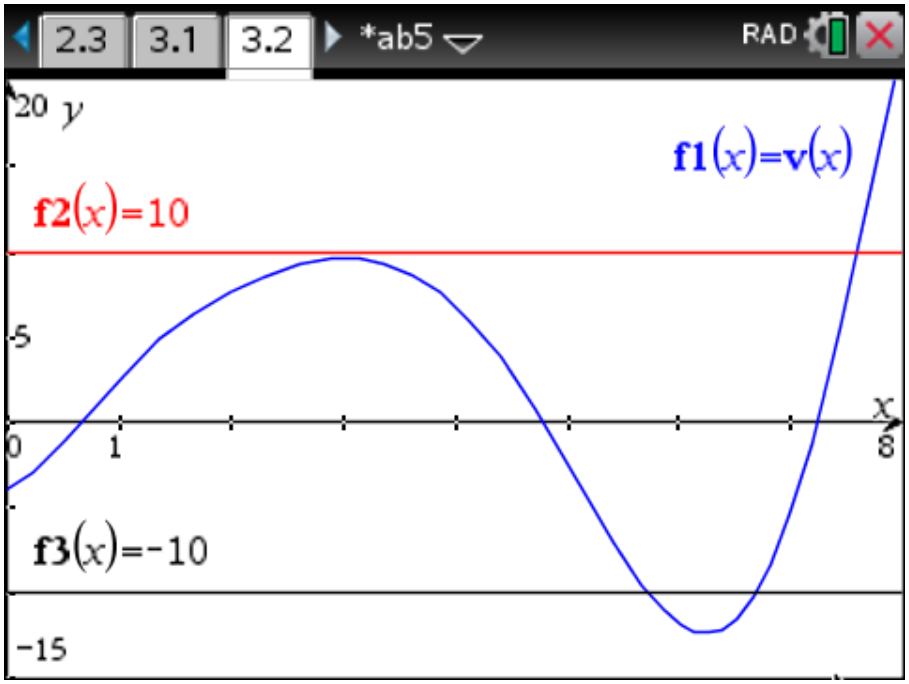
(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

Example

Solution

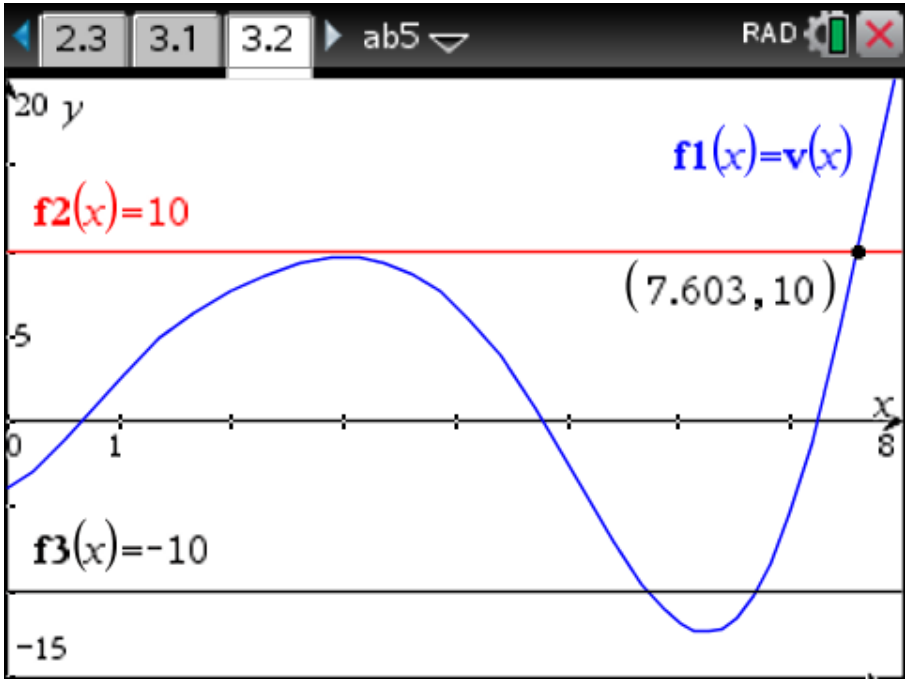
(a) The speed of the particle is the absolute value of the velocity.

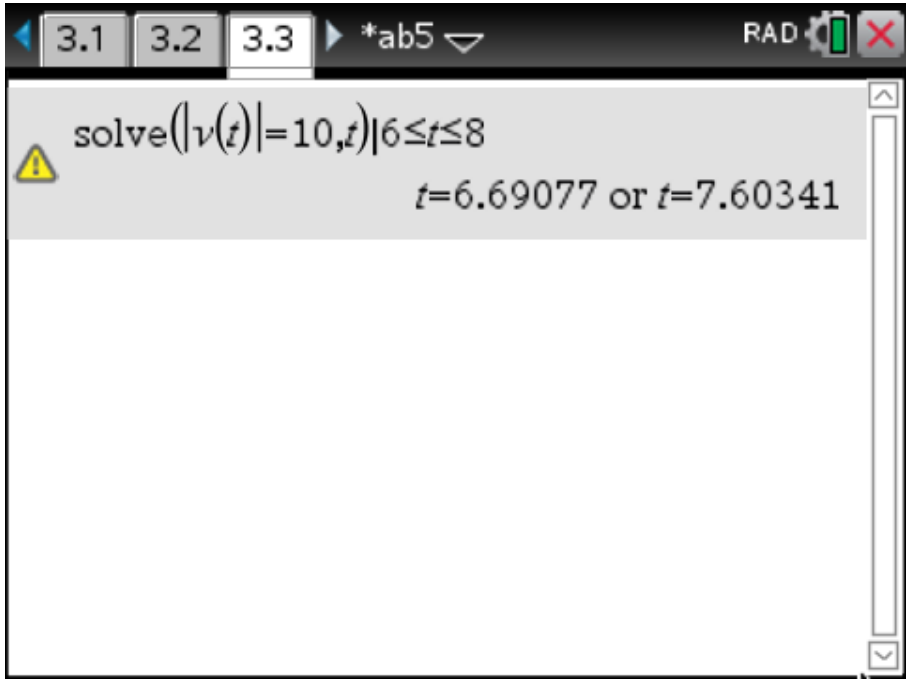
Solve the expression $|v(t)| = 10$ on the interval $6 \leq t \leq 8$.



Example

Solution





The speed of the particle is 10 at $t = 6.691$ and at $t = 7.603$.

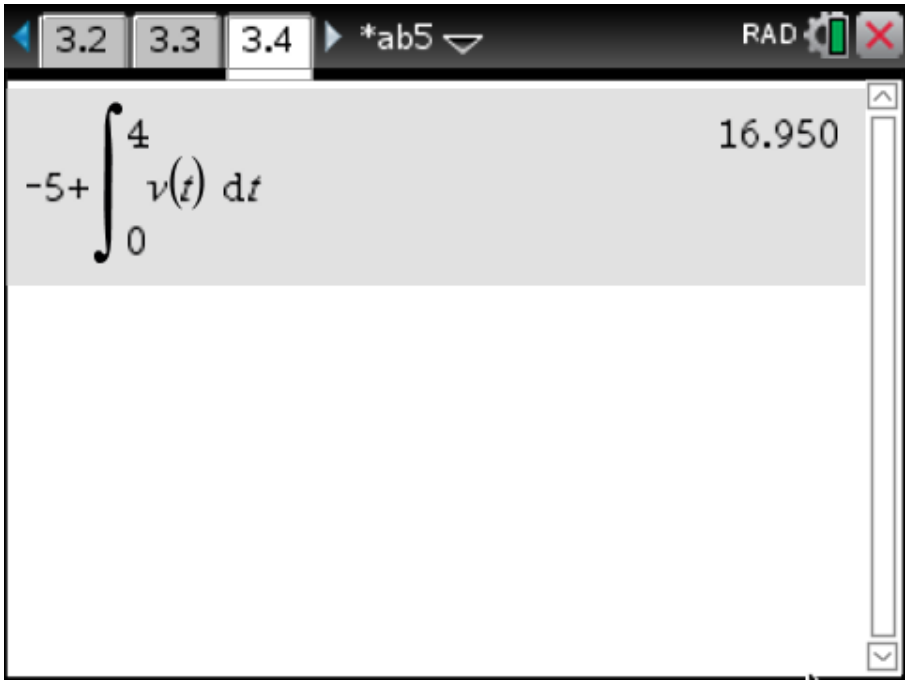
Example

Solution

(b) $s(t) = s(0) + \int_0^t v(x)\,dx = -5 + \int_0^t v(x)\,dx$

The position of the particle at time $t = 4$:

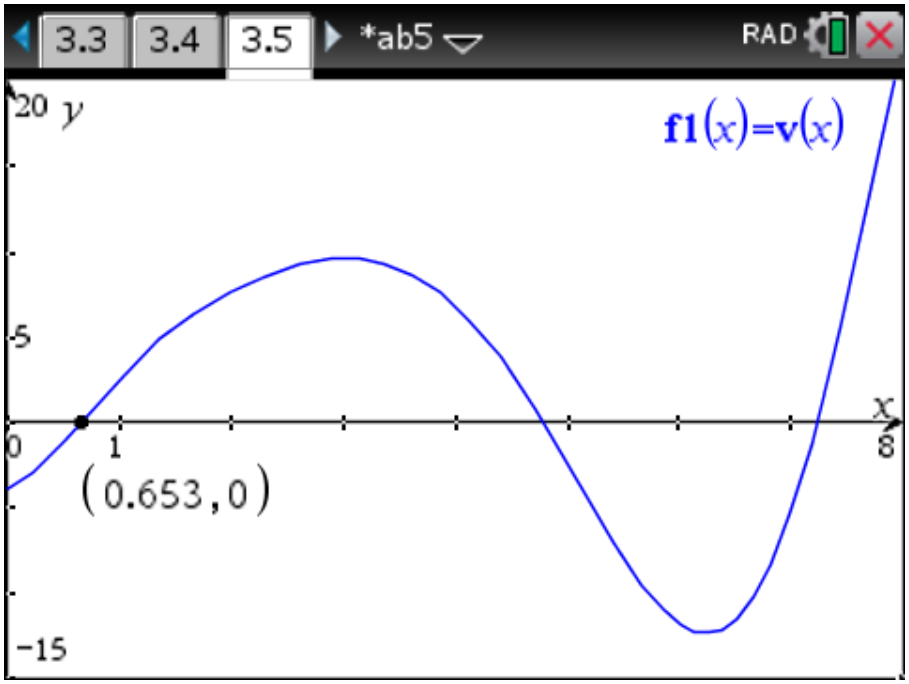
$$s(4) = -5 + \int_0^4 v(x)\,dx = 16.95$$

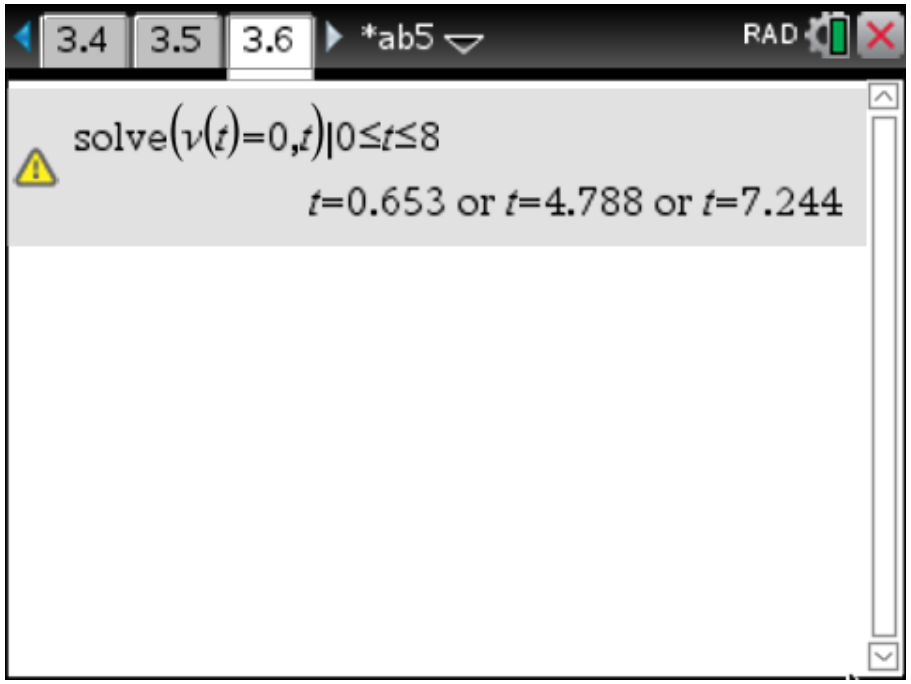


Example

Solution

(c) Use the graph of v to find the values of t where $v(t)$ changes sign.





$v(t) = 0$ when $t = 0.653,\ 4.788,\ 7.244$

$v(t)$ changes sign from negative to positive at times $t = 0.653,\ 7.244$

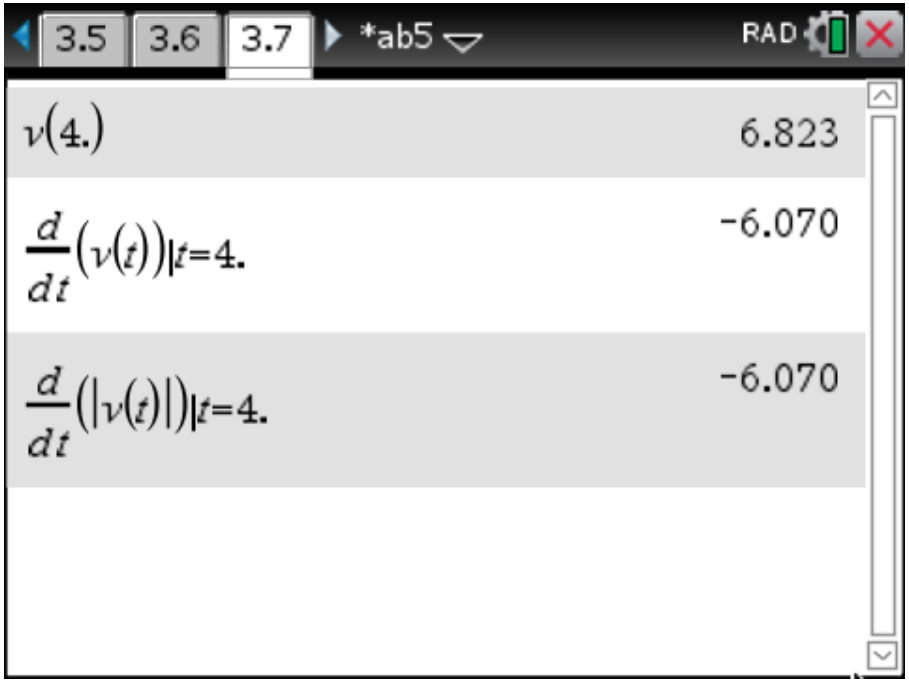
$v(t)$ changes sign from positive to negative at time $t = 4.788$.

The particle changes direction at times $t = 0.653,\ 4.788,\ 7.244$.

Example

Solution

(d) Consider $v(4)$ and $a(4)$.

$$v(4) = 6.823 > 0, \quad a(4) = -6.070 < 0.$$

The speed of the particle is decreasing at time $t = 4$ because the velcotiy and the acceleration have different signs.