TI in Focus: AP® Calculus

2020 Mock AP® Calculus Exam

AB-1: Solutions, Concepts, and Scoring Guidelines

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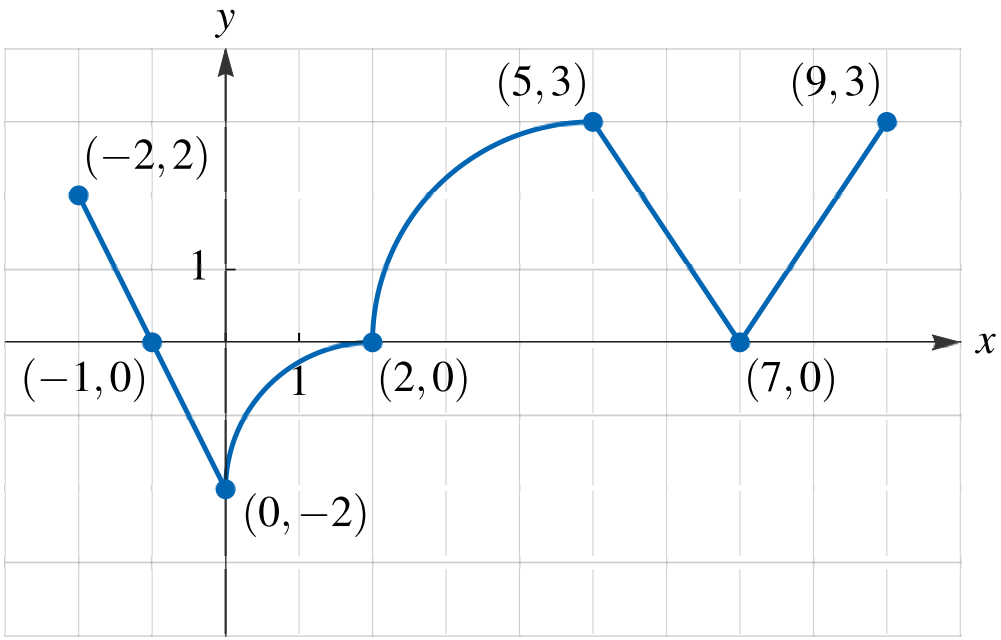
AB 1

The continuous function f has domain $-2 \leq x \leq 9$. The graph of f, consisting of three line segments and two quarter circles, is shown in the figure.



Graph of f

Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$ for $-2 \leq x \leq 9$.

(c) For $-2 \leq x \leq 9$, on what open intervals is g increasing and concave down? Give a reason for your answer.

Key Concepts

Definition

A function f is **increasing** on an interval I if

$$f(x_1) < f(x_2) \quad \text{whenever} \quad x_1 < x_1 \text{ in } I$$

The function is **decreasing** on I if

$$f(x_1) < f(x_2) \quad \text{whenever} \quad x_1 < x_2 \text{ in } I$$

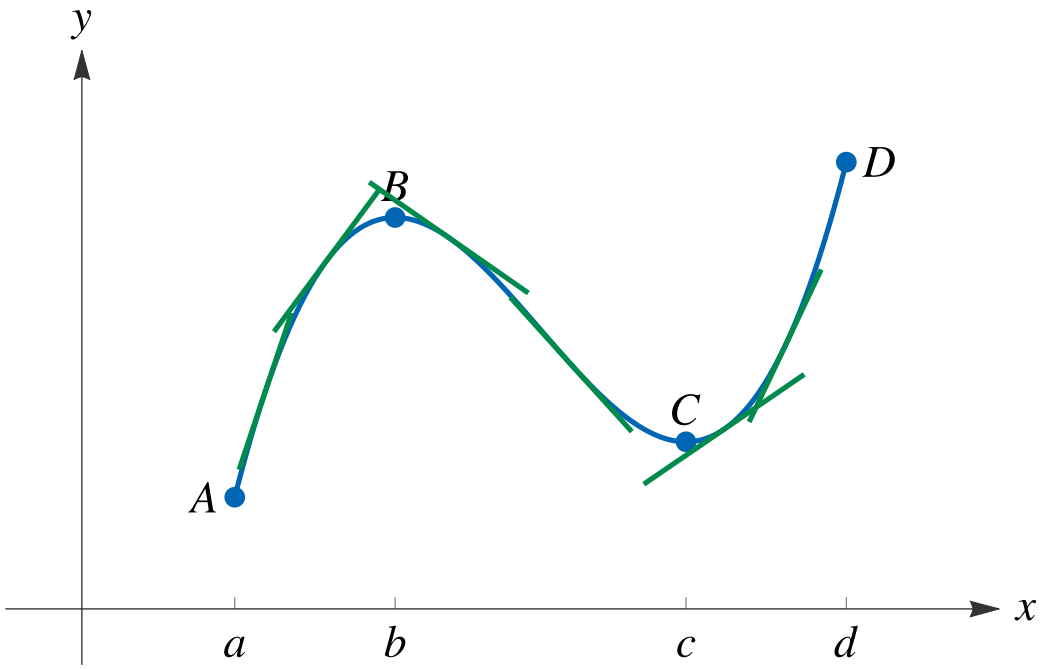
Increasing/Decreasing Test (I/D Test)

(a) If $f'(x) > 0$ on an interval, then f is increasing on that interval.

(b) If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

A Closer Look

1. Consider the figure.



Between a and b, and between c and d:
tangent lines have positive slopes, and $f'(x) > 0$.

Between b and c:
tangent lines have negative slopes, and $f'(x) < 0$.

2. Proof: Mean Value Theorem

Definition

Let f be a differentiable function.

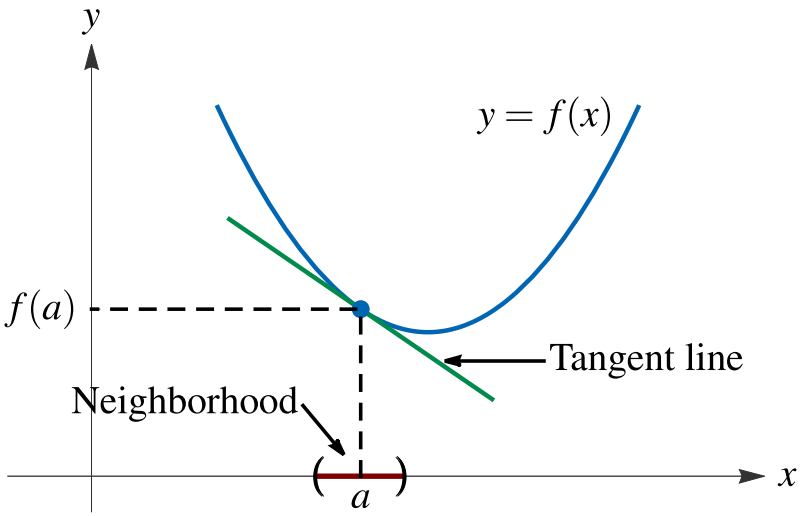
f is **concave up** at a if the graph of f is above the tangent line to f at a for all x in a neighborhood containing a (but not equal to a).

f is **concave down** at a if the graph of f is below the tangent line to f at a for all x in a neighborhood containing a (but not equal to a).

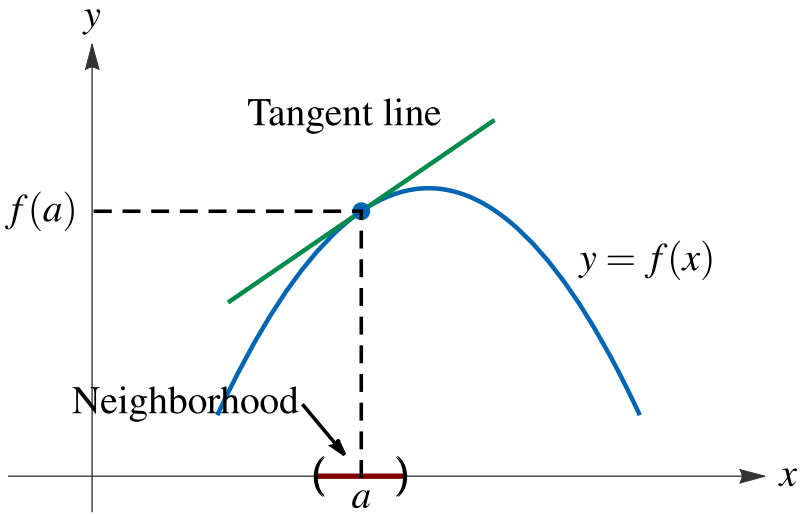
A Closer Look

1. Neighborhood: an open interval containing a.
2. Concavity is defined in terms of a single number, not an interval.

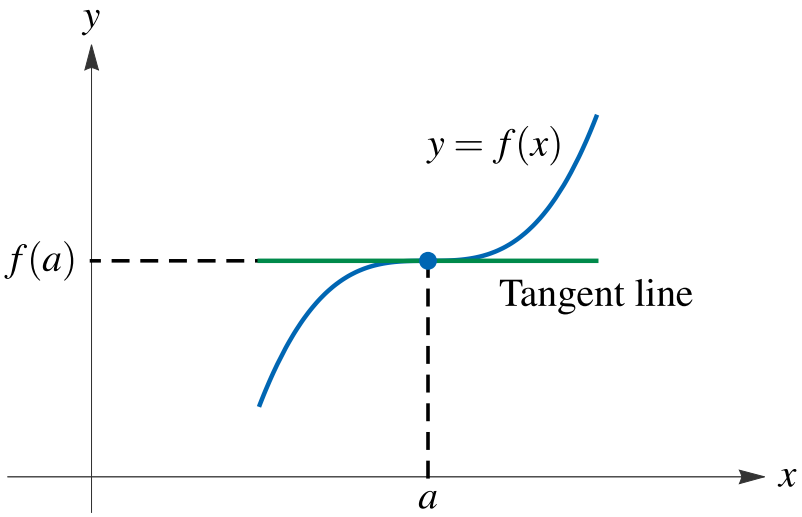
   The graph of a function can be concave up, concave down, or have no concavity at a number.



The graph of f is concave up at a. The graph of f lies above the tangent line in a neighborhood of a.



The graph of f is concave down at a. The graph of f lies below the tangent line in a neighborhood of a.



The graph of f has no concavity at a.

Concavity Test

(a) If $f''(x) > 0$ for all x in I, then the graph of f is concave up on I.

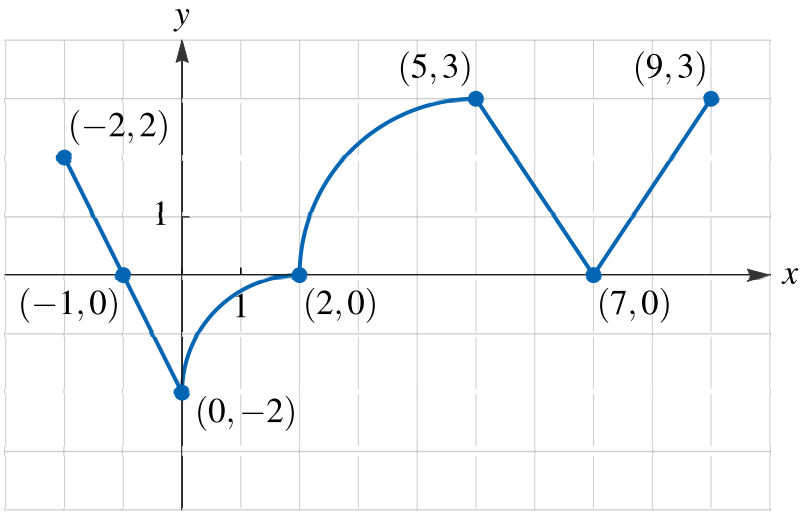
(b) If $f''(x) < 0$ for all x in I, then the graph of f is concave down on I.

A Closer Look

1. The Concavity Test allows us to draw a conclusion about the behavior of the graph of f if $f''(x) > 0$ or $f''(x) < 0$.

   Need to examine numbers a where $f''(a) = 0$ or $f''(a)$ DNE.
2. The graph of f is either concave up, concave down, or has no concavity at a number.
3. The point on the graph of f where it changes concavity is called an inflection point.

Solution



g is increasing where $g' = f$ is positive: $(-2, -1), \quad (2, 7), \quad (7, 9)$

g is concave down where $g' = f$ is decreasing: $(-2, 0), \quad (5, 7)$

g is increasing and concave down on the intervals: $(-2, -1)$ and $(5, 7)$.

Scoring Guidelines

(c) g is increasing where $g' = f$ is positive.

g is concave down where $g' = f$ is decreasing.

g is increasing and concave down on the intervals $(-2, -1)$ and $(5, 7)$.

$$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$$

Scoring Notes

- The first point is earned for the correct intervals: open, closed, half-open.
- To be eligible for the second point: must have earned the first point.
- A reason referencing only g' is eligible for the second point.
- A reason referencing only f is not eligible for the second point unless the student has made the connection $g' = f$.
- The reason g' is decreasing can be expressed as: g'' is negative, or the slope of g' is negative.
- A response referencing *the function*, *the slope*, or *it* are too vague.

(d) Find the value of $g(-1)$. Show the computations that lead to your answer.

Key Concepts

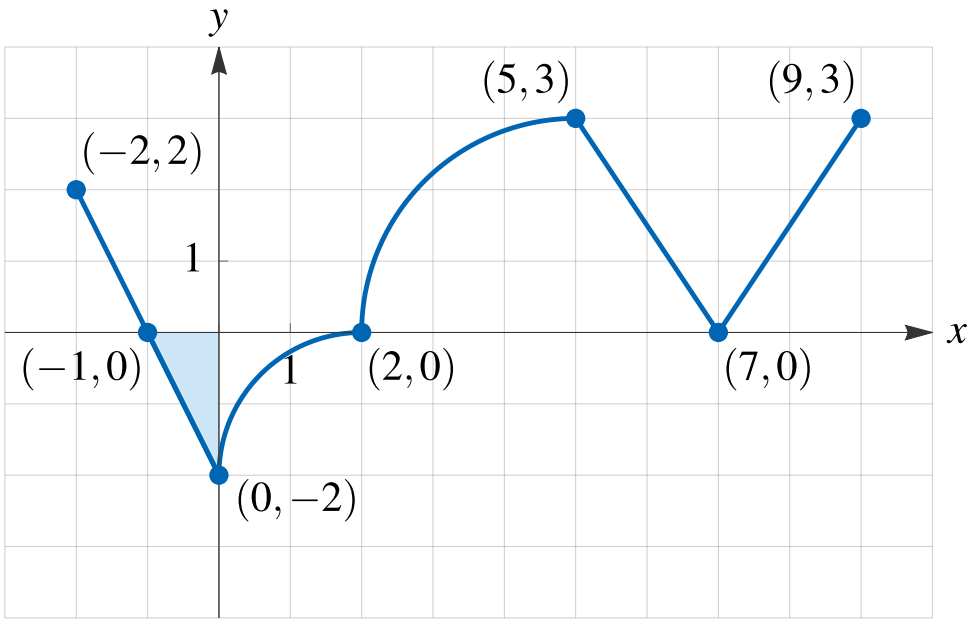
1. The definite integral can be interpreted as net area.

$$\int_a^b f(x)\,dx = A_1 - A_2$$

where A_1 is the area of the region above the x-axis and below the graph of f, and A_2 is the area of the region below the x-axis and above the graph of f.

2. $\displaystyle\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

Solution



$$g(-1) = \int_0^{-1} f(t)\,dt$$ Definition of the function g.

$$= -\int_{-1}^{0} f(t)\,dt$$ Property of definite integrals.

$$= -\left(-\frac{1}{2}(1)(2)\right) = 1$$ Area of a triangle.

Scoring Guidelines

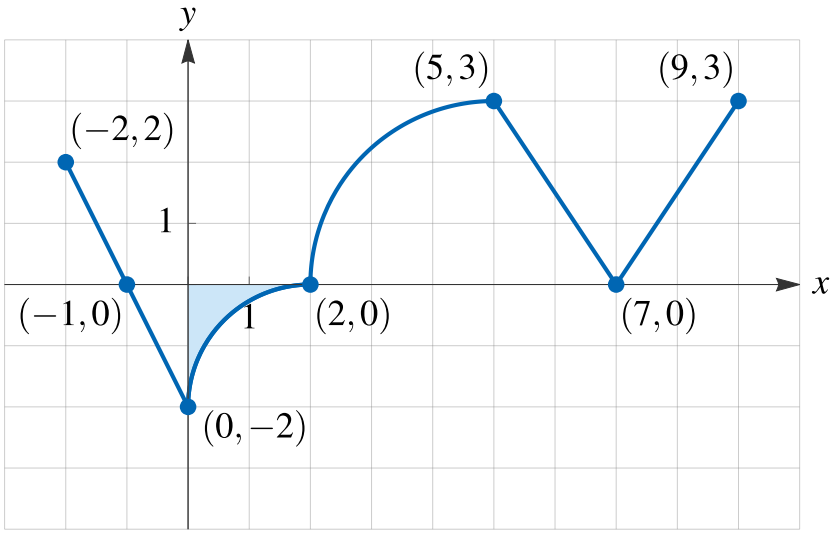
(d) $g(-1) = \int_0^{-1} f(t)\,dt$ $= -\int_{-1}^{0} f(t)\,dt = -\left(-\frac{1}{2}(1)(2)\right) = 1$	1 : answer

Scoring Notes

- Correct supporting work is needed to earn this point.
- A bald 1 does not earn this point.

(e) Find the value of $g(2)$. Show the computations that lead to your answer.

Solution



$$g(2) = \int_0^2 f(t)\,dt$$ Definition of the funciton g.

$$= -\left(2 \cdot 2 - \frac{1}{4} \cdot \pi \cdot 2^2\right)$$ Area of a square minus area of a quarter circle.

$$= -(4 - \pi) = \pi - 4$$ Simplify.

Scoring Guidelines

(e) $g(2) = \int_0^2 f(t)\,dt$

$$= -\left(2 \cdot 2 - \frac{1}{4} \cdot \pi \cdot 2^2\right) = -(4 - \pi)$$

$$2 : \begin{cases} 1 : \text{area of quarter circle} \\ 1 : \text{answer} \end{cases}$$

Scoring Notes

- The response does not need to include a definite integral.
- $\pi - 4$ with no supporting work earns one point.
- Evidence of area of quarter circle in calculation earns one point.

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