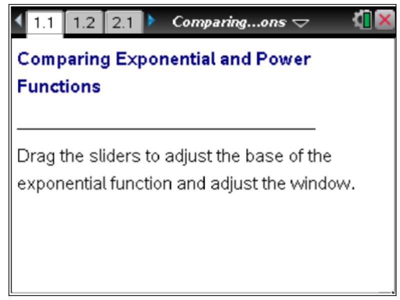
Comparing Exponential and Power Functions

MATH NSPIRED



Math Objectives

- Students will be able to use various graphical representations to determine which of two functions is greater for large values of *x.*
- Students will understand that any exponential function of base greater than 1 will be greater than any power function for sufficiently large values of *x.*
- Students will be able to determine which of two functions is greater given a graph of the ratio of the two functions.
- Students will be able to construct arguments based on graphs and other reasoning to support the conclusion that exponential functions exceed power functions for sufficiently large values of *x*.
- Students will be able to reason abstractly and quantitatively (CCSS Mathematical Practice).

Vocabulary

- exponential function
- power function
- base

About the Lesson

- This lesson involves opportunities to make and test conjectures about the relative size and growth of power functions and exponential functions using graphical representations.
- As a results, students will:
  - Use sliders to adjust the base of an exponential function and compare the varying exponential functions to x^2 and x^3.
  - Adjust the graphical viewing window to observe that for any base greater than 1, the exponential function eventually overtakes both power functions.
  - Adjust the base of the exponential function and the power of the power function, observe the graph of the two functions, and observe the graph of their ratio to make determinations about relative size.

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- Use **Quick Poll** to assess student understanding
- Use **Class Capture** to exhibit different exponential functions.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Comparing_Exponential_and_Power_Functions_Student.doc
- Comparing_Exponential_and_Power_Functions_Student.pdf

TI-Nspire document

- Comparing_Exponential_and_Power_Functions.tns

Discussion Points and Possible Answers

Tech Tip: Use the sliders to change the values of *r* and *b* and the maximum *x*-value displayed in the window.

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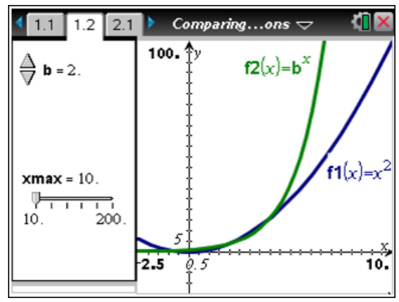

1. Use the arrow labeled **b** to adjust the base of the exponential function to be 2, so you have the functions x^2 and 2^x.
   a. On what interval(s) is $x^2 > 2^x$? How do you know? Did you need the graph to determine this?

   Answer: On approximately (-0.7666, 2) and (4, ∞). Students can determine from the graph, by approximating or finding intersection points, or numerically.

   b. Use the arrow to increase the base of the exponential function. For bases larger than 2, which is greater in the long run, 2^x or x^2 How do you know?

   Answer: The value of 2^x is greater for larger values of *x*. Students might reason from graphical observations. Alternatively, students might observe that for bases larger than 2, and for positive values of *x*, $b^x > 2^x$ and thus by transitivity, $2^x > x^2$ in the long run.

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See Note 1 at the end of this lesson.

2. Use the arrow labeled **b** to adjust the base of the exponential function to be less than 2.
   a. Are there any values of *b* for which the exponential function seems to be less than x^2 in the long run? What seems to be happening with these functions?

   Sample Answers: Students might observe that values of *b* between 1 and 2 appear to result in the power function overtaking the exponential function over time.

b. Choose a value of b other than 1 so that x^2 appears to be greater than b^x in the long run. Now use the other slider to change **xmax** in the graphing window. What appears to happen?

Answer: Changing the window reveals that the exponential overtakes x^2 at some point.

> **Teacher Tip:** Discuss with students the change in the window. Why does the appearance of b^x change, but the appearance of x^2 does not? This is because the range of y-values displayed is set to be the square of the range of x-values displayed. Thus the proportions are maintained, keeping the appearance of x^2 consistent.

TI-Nspire™ Navigator™ Opportunity: *Class Capture*
See Note 2 at the end of this lesson.

c. Why couldn't you choose $b = 1$ in part b? What would have happened?

Answer: When $b = 1$, the function is no longer exponential. It is a constant function, so after $x = 1$, x^2 would always be greater.

d. Based on your observations, for any $b > 1$, what appears to be greater in the long run, b^x or x^2? Explain.

Answer: The value of b^x appears to be greater. It always eventually overtakes x^2, as seen in the graph.

e. How do you know that x^2 won't be greater than b^x at some point later in the graph?

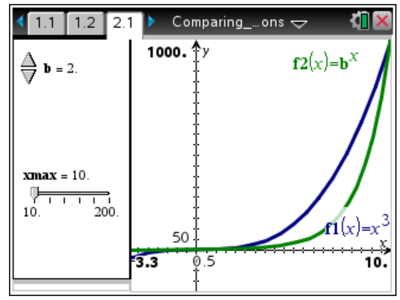
Sample Answers: Students might develop arguments focused on the respective growth rates of the two functions.

f. Do you think this result would be different if you were using the power function x^3 instead of x^2? Explain your thinking.

Sample Answers: The correct answer is no. However, students will likely guess that it would be different.

TI-Nspire™ Navigator™ Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

Move to page 2.1.



3. Use the arrow labeled **b** to change the base of the exponential.
    a. For what values of *b* does b^x appear to be greater than x^3 in the long run? What did you observe from the graph to lead you to this conclusion?

    Sample Answers: The value of b^x appears greater for values of *b* that are larger than 2. For all of these, the end of the exponential function seems to be greater than the end of x^3.

    b. Choose a small value of *b* (for example, 1.1), and use the **xmax** slider to adjust the window. What do you observe?

    Answer: When **xmax** is increased, it becomes clear that any exponential with base > 1 will overtake x^3 as *x* gets larger.

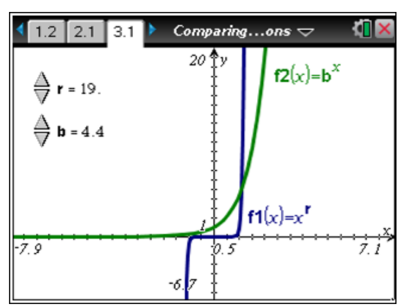
    c. For what values of *b* will b^x be greater than x^3 in the long run?

    Answer: The value of b^x will be greater for any values of *b* greater than 1.

    d. Do you believe this result will hold true for all power functions? For what values of *b* and *r* do you think b^x will be greater than x^r in the long run?

    Sample Answer: Yes. For any *b* and *r* greater than 1, b^x will be greater than x^r in the long run.

Move to page 3.1.



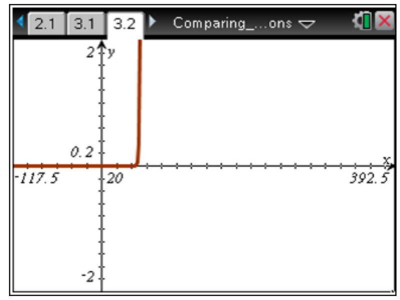
4. Use the arrows to adjust the values of *b* and *r*. For what values of *b* and *r* does b^x appear to be greater than x^r in the long run? How does the graph help you to determine this?

    Sample Answers: The value of b^x appears greater when *r* is not much greater than *b*. Students might provide some range of values. The graph helps to determine this because it shows the graphs of each function, making it possible to see when one is greater than the other.

Adjust *b* and *r* so that x^r appears to be greater than b^x for large values of *x*.

Teacher Tip: Students might need some guidance in examining different values of *r* and *b*. Teachers might want to provide sets of values to test.

Move to page 3.2.



5. The function graphed on page 3.2 is the ratio $\frac{b^x}{x^r}$ for the *b* and *r* you set on Page 3.1.

   a. For which values of $\frac{b^x}{x^r}$ is $b^x > x^r$? How do you know?

   Answer: When $\frac{b^x}{x^r} > 1$, $b^x > x^r$, since the numerator of a fraction is greater than the denominator when the value of the fraction is greater than 1.

   b. What does the graph of $\frac{b^x}{x^r}$ tell you about the relationship between b^x and x^r for large values of *x*? Explain.

   Answer: It allows you to determine when $b^x > x^r$ in a larger window. When the graph of the ratio is greater than 1, we know that $b^x > x^r$. This shows that even for *b* very close to 1 and large *r*, we can see that eventually that ratio is larger than 1, so eventually $b^x > x^r$.

   c. Go back to Page 3.1, adjust the values of *b* and *r*, and return to Page 3.2 to observe the changes in the graph of the ratio. You might want to do this several times. Based on your observations, for which values of *b* and *r* will b^x be greater than x^r for large values of *x*? Explain your reasoning.

   Answer: For any value of $b > 1$ and any value of *r*, b^x will be greater than x^r for large values of *x*. Reasoning will vary, but students might note that the exponential is eventually growing at a faster rate than the power function. This can be seen through the ratio, as the ratio function grows at an increasing rate, indicating that the growth of the exponential numerator is overpowering the growth of the power function denominator.

Teacher Tip: You might have students graph **f**(*x*) = 1 on page 3.2 so that they can see graphically when the ratio is less than or greater than 1.

TI-Nspire™ Navigator™ Opportunity: ***Class Capture***
See Note 4 at the end of this lesson.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- Any exponential function with base greater than 1 will eventually overtake any power function.
- A function that is the ratio of two functions can be used to better understand the difference in growth rates of the individual functions.

Assessment

Provide power functions and exponential functions to compare for large values of x. Include functions with powers and bases smaller than 1. Ask students to predict and use the calculator to test predictions about when an exponential with a small base will overtake a power function with a larger power.

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Note 1

Question 1, Class Capture: Students are comparing x^2 to b^x. Capture and share student screens displaying different values of b, and encourage students to use the variety of screens to make conjectures.

Note 2

Question 2b, Class Capture: Students are changing the viewing window to show that for all $b > 1$, b^x is eventually greater than x^2. Capture and share several student screens to emphasize that this will be the case for all $b > 1$.

Note 3

Question 2f, Quick Poll: Use the **Quick Poll** tool to determine which portion of the class believes x^3 will be larger than some exponential functions and which believes that an exponential will always exceed x^3. Teachers might want to give students a chance to discuss and share their reasoning.

Note 4

Question 5, Class Capture: There are opportunities to capture both screen shots of students' exponential and power functions together and screen shots of the ratio functions. Teachers might want to display a pair of functions and their associated ratio function together; several of the exponential/power pairs; or several of the ratios.