**Concepts**

If a function  has derivatives of all orders, then under certain conditions we can write  as a Taylor series centered at :



In the case where , the Taylor series becomes



This is called a Maclaurin series.

This expression means that  is the limit of the sequence of partial sums. The partial sums are



The expression for  is a polynomial of degree .  is called the th-degree Taylor polynomial of  at , or centered at .

**Course and Exam Description Unit**

Section 10.11: Finding Taylor Polynomial Approximations of Functions

Section 10.14: Finding Taylor or Maclaurin Series for a Function

**Calculator Files**

Taylor\_Polynomials\_CAS.tns

**Using the Document**

Taylor\_Polynomials\_CAS: This calculator file provides a tool for generating and graphing Taylor polynomials. The degree of the Taylor polynomial is changed using the arrow clicker for , and the value for  can be changed by dragging the point on the -axis or by entering a new -coordinate in the ordered pair displayed on the graph screen.

Page 1.1

|  |  |
| --- | --- |
|  | This introductory screen provides information for constructing and graphing a Taylor polynomial for a function . This function, , is defined on Page 1.2, a Graphs page. Once the function is defined, the degree of the Taylor polynomial is set using the arrow clicker. The value of , the center of the Taylor polynomial, is set by moving the corresponding point on the -axis or by entering a new -coordinate in the ordered pair displayed on the graph screen. |

Page 1.2

|  |  |
| --- | --- |
|  | The default settings are , , and . The function  is plotted as a dotted curve, and the Taylor polynomial, , is plotted in blue.  Change the value of  by using the arrow clicker, and change the value of  by dragging the corresponding point on the -axis or by entering a new -coordinate in the ordered pair displayed on the graph screen. |

Page 1.3

|  |  |
| --- | --- |
|  | The function  is defined to be the expression for the th-degree Taylor polynomial centered at . The polynomial is displayed by the second Math Box. The last Math Box is used to evaluate this Taylor polynomial at a specific value. |

**Suggested Applications and Extensions**

1. (a) Find the Taylor polynomials up to degree  for  centered at .

Examine these graphs as  increases.

(b) Evaluate  and these Taylor polynomials at , and .

(c) Explain how the Taylor polynomials converge to .

1. Find the Taylor polynomial  for the function  centered at the number . Observe how the graphs of the Taylor polynomials change as  increases, and find an interval in which the Taylor polynomial is a good approximation to .
2. , 
3. , 
4. , 
5. , 
6. , 
7. , 
8. Find the Taylor polynomial  for the function  centered at 0. Observed how the graphs change as  increases, find an interval in which the Taylor polynomial is a good approximation to , and find .
9.  
10.  
11. , 
12. , 
13. , 
14. , 
15. Find the Taylor polynomial  for the function  centered at . Explain this result.
16. (a) Find the Taylor polynomial  for the function  centered at .

(b) Find the Taylor polynomial  for the function  centered at .

(c) Find the Taylor polynomial  for the function  centered at .

Explain how this Taylor polynomial is related to those found in parts (a) and (b).