Exploring Limits of a Sequence and Sum of a Series

Using the Document

Sequences&Series.tns: This calculator file provides a technology tool for investigating the limit of an arbitrary sequence $\{a_n\}$ and whether an infinite series of the form $\sum_{k=1}^{\infty} a_k$ is convergent or divergent. A slider is used to display values of a_n and the partial sums $\sum_{k=1}^{n} a_k$ for various values of n. A table of these values is automatically computed and displayed in a Lists and Spreadsheet page.

The default sequence is $a_n = \dfrac{1}{n^3}$ and the corresponding series is $\sum_{k=1}^{\infty} \dfrac{1}{k^3}$.
The values for n used in this file are $n = 1, 2, 3, 4, 5, 10, 100, 1000, 10000$.

Suggested Applications and Extensions

Find several values of each sequence. Use these values to conjecture if the sequence converges or diverges. If you think it converges, guess the limit.

1. $a_n = \dfrac{7 - 5n^2}{3 + 10n}$
2. $a_n = \left(\dfrac{1}{e}\right)^n$
3. $a_n = \dfrac{n}{e^n}$
4. $a_n = \dfrac{\ln n}{n}$
5. $a_n = \dfrac{n^n}{n!}$
6. $a_n = \dfrac{\cos n}{n}$
7. $a_n = \left(3 + \dfrac{3}{n}\right)^n$
8. $a_n = \dfrac{\sin(n\pi)}{3^n}$
9. $a_n = \sqrt[n]{2^n + 3^n}$
10. $a_n = \tan^{-1}\left(\dfrac{-n^2}{n^2 - 7}\right)$
11. $a_n = \ln(n) - \ln(n + 1)$
12. $a_n = e^{1/\sqrt{n}}$

Find several partial sums for each series. Use these values to guess whether the series is convergent or divergent.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{5}{n^2 + n}$
2. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5n^2 - n + 3}$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{e^n}$
5. $\displaystyle\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^2}$
6. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$
7. $\displaystyle\sum_{n=1}^{\infty} \cos n$
8. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} e^{3/n}$
9. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{2n^3 + n^2 - 7n + 5}$
10. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n}{n!}$

Extended Application Questions

1. Determine whether there is a relationship between series convergence and the terms of the corresponding sequence. Are there any general sequences $\{a_n\}$ such that the corresponding series $\sum_{n=1}^{\infty} a_n$ is guaranteed to converge? Diverge?

2. In those series that contain some terms that are positive and some terms that are negative, consider the series of the absolute value of each term, that is, $\sum_{n=1}^{\infty}|a_n|$. Is there a relationship between the convergence or divergence of $\sum_{n=1}^{\infty}|a_n|$ and the convergence or divergence of $\sum_{n=1}^{\infty}a_n$?