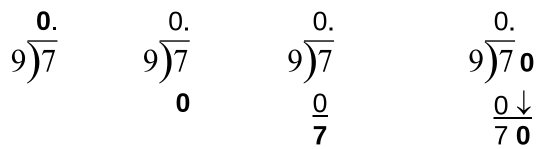
7 ÷ 9 Computed with the Division Algorithm

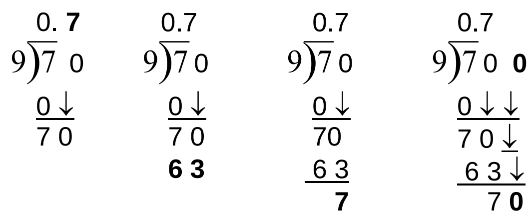
Demonstrate this algorithm to your students and point out that the repeating pattern of digits comes from the remainder found after each digit in the quotient is found.

Divide Multiply Subtract Append the zero and bring it down



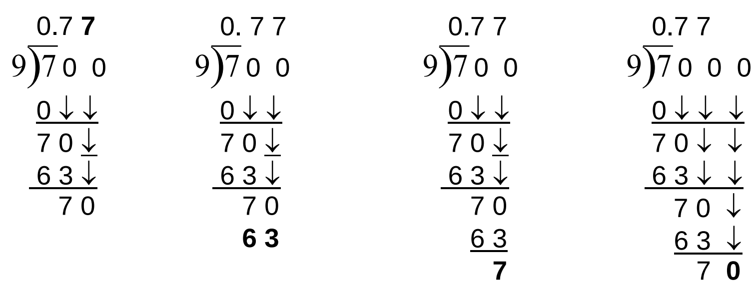
7 appears as a remainder

Divide Multiply Subtract Append the zero and bring it down



7 appears as a remainder again

Divide Multiply Subtract Append the zero and bring it down



7 appears as a remainder again

The algorithm can be used to find more digits in the quotient. Each of those digits will be a “7”.

23 ÷ 99 Computed with the Division Algorithm

Demonstrate this algorithm to your students and point out that the repeating pattern of digits comes from the remainder found after each digit in the quotient is found.

Divide

$$\begin{array}{r} \mathbf{0}. \\ 99\overline{)23} \end{array}$$

Multiply

$$\begin{array}{r} 0. \\ 99\overline{)23} \\ \mathbf{0} \end{array}$$

Subtract

$$\begin{array}{r} 0. \\ 99\overline{)23} \\ \underline{0} \\ \mathbf{23} \end{array}$$

Append the zero and bring it down

$$\begin{array}{r} 0. \\ 99\overline{)23\,\mathbf{0}} \\ \underline{0\downarrow} \\ 23\,\mathbf{0} \end{array}$$

23 appears as a remainder

Divide

$$\begin{array}{r} 0.2 \\ 99\overline{)23\,0} \\ \underline{0\downarrow} \\ 23\,\mathbf{0} \end{array}$$

Multiply

$$\begin{array}{r} 0.2 \\ 99\overline{)23\,0} \\ \underline{0\downarrow} \\ 23\,0 \\ 19\,8 \end{array}$$

Subtract

$$\begin{array}{r} 0.2 \\ 99\overline{)23\,0} \\ \underline{0\downarrow} \\ 23\,0 \\ \underline{19\,8} \\ \mathbf{3\,2} \end{array}$$

Append the zero and bring it down

$$\begin{array}{r} 0.2 \\ 99\overline{)23\,0\,0} \\ \underline{0\downarrow\downarrow} \\ 230\downarrow \\ \underline{198\downarrow} \\ 32\,\mathbf{0} \end{array}$$

32 appears as a remainder

Divide

$$\begin{array}{r} 0.2\,\mathbf{3} \\ 99\overline{)23\,0\,0} \\ \underline{0\downarrow\downarrow} \\ 230\downarrow \\ \underline{198\downarrow} \\ 32\,0 \end{array}$$

Multiply

$$\begin{array}{r} 0.2\,3 \\ 99\overline{)23\,0\,0} \\ \underline{0\downarrow\downarrow} \\ 230\downarrow \\ \underline{198\downarrow} \\ 32\,0 \\ \mathbf{29\,7} \end{array}$$

Subtract

$$\begin{array}{r} 0.2\,3 \\ 99\overline{)23\,0\,0} \\ \underline{0\downarrow\downarrow} \\ 230\downarrow \\ \underline{198\downarrow} \\ 32\,0 \\ \underline{29\,7} \\ \mathbf{2\,3} \end{array}$$

Append the zero and bring it down

$$\begin{array}{r} 0.2\,3 \\ 99\overline{)23\,0\,0\,\,0} \\ \underline{0\downarrow\downarrow\downarrow} \\ 230\downarrow\downarrow \\ \underline{198\downarrow\downarrow} \\ 32\,0\downarrow \\ \underline{29\,7\downarrow} \\ 2\,3\,\mathbf{0} \end{array}$$

23 appears as a remainder again

The algorithm can be used to find more digits in the quotient. The digits “2” and “3” will alternate.

Algebraic Approach

Though your students will not know enough Algebra, you might recall that Algebra is another approach to find fractions for repeating decimals like 0.888… . This approach may not be appropriate for most Middle School students but is one way to show how to use Algebra to demonstrate that repeating decimals can be changed into fractions.

$$\begin{aligned}
\text{So let } x &= 0.888\ldots \\[1em]
\text{Multiply } x \text{ by ten to get: } 10x &= 8.888\ldots \\
\underline{\text{Subtract 1 times } x \;-1x} &\underline{= 0.888\ldots} \\
9x &= 8.000\ldots \\
x &= 8/9
\end{aligned}$$

The choice of multipliers 1 and 10 were made to have the subtraction yield zeroes after the decimal.

If we have a decimal like 0.565656… we do the following:

$$\begin{aligned}
\text{So let } x &= 0.565656\ldots \\[1em]
\text{Multiply } x \text{ by 100 to get: } 100x &= 56.565656\ldots \\
\underline{\text{Subtract 1 times } x \quad -1x} &\underline{= \;\; 0.565656\ldots} \\
99x &= 56.000\ldots \\
x &= 56/99
\end{aligned}$$

The choice of multipliers 1 and 100 were made to have the subtraction yield zeroes after the decimal.

If we have a decimal like 0.1565656… we do the following:

$$\begin{aligned}
\text{So let } x &= 0.1565656\ldots \\[1em]
\text{Multiply } x \text{ by 1000 to get: } 1000x &= 156.565656\ldots \\
\underline{\text{Subtract 10 times } x \quad -10x} &\underline{= \quad 1.565656\ldots} \\
990x &= 155.000\ldots \\
x &= 155/990 \\
\text{Reducing the fraction yields: } x &= 26/195
\end{aligned}$$

The choice of multipliers 1000 and 10 were made to have the subtraction yield zeroes after the decimal.

Decomposition Approach

Your students are likely to be experienced adding two decimals but not familiar with writing a decimal as the sum of two decimals — a decomposition Your students are likely to be familiar with finding common denominators and multiplying or dividing by powers of ten, so this approach might be useful to your students. They are also not likely to be experienced writing a fraction as a product of two fractions—another kind of decomposition.

Using of Powers of Ten and Decomposition by Multiplication:

1. Since $\frac{7}{9}=0.\overline{7}$ we can factor the fraction into a product of two fractions:
   $\frac{7}{90}=\frac{1}{10}\times\frac{7}{9}=0.0\overline{7}$ The decimal is moved to reflect the multiplication by $\frac{1}{10}$.

2. This can lead us to recognize that $\frac{7}{900}=\frac{1}{100}\times\frac{7}{9}=0.00\overline{7}$ by factoring the fraction's denominator.

3. We can also decompose the fraction $\frac{56}{990}$ as follows: $\frac{56}{990}=\frac{1}{10}\times\frac{56}{99}=0.0\overline{56}$. The decimal is moved to reflect the multiplication by $\frac{1}{10}$.

Using of Powers of Ten and Decomposition by Addition:

1. We can decompose the decimal into a sum of two decimals: $0.1\overline{56}=0.1+0.0\overline{56}$.
   Next, the sum of decimals is changed to a sum of fractions: $0.1+0.0\overline{56}=\frac{1}{10}+\frac{56}{990}$.
   The second decimal comes from the example 3. This sum of two fractions can be computed when the fractions have a common denominator: $\frac{1}{10}+\frac{56}{990}=\left(\frac{1}{10}\times\frac{99}{99}\right)+\frac{56}{990}$
   The addition continues: $\left(\frac{1}{10}\times\frac{99}{99}\right)+\frac{56}{990}=\frac{99}{990}+\frac{56}{990}$.
   Finally, leading to: $\frac{99}{990}+\frac{56}{990}=\frac{99+56}{990}=\frac{155}{990}$.

2. We can decompose the decimal into a sum of two decimals: $0.12\overline{56}=0.12+0.00\overline{56}$.
   Next, the sum of decimals is changed to a sum of fractions: $0.12+0.00\overline{56}=\frac{12}{100}+\frac{56}{9900}$.
   The second decimal comes from the example 3 above. This sum of two fractions can be computed when the fractions have a common denominator:
   $\frac{12}{100}+\frac{56}{9900}=\left(\frac{12}{100}\times\frac{99}{99}\right)+\frac{56}{9900}$ The addition continues:
   $\left(\frac{12}{100}\times\frac{99}{99}\right)+\frac{56}{9900}=\frac{1188}{9900}+\frac{56}{9900}$. At last we see: $\frac{1188}{9900}+\frac{56}{9900}=\frac{1188+56}{9900}=\frac{1244}{9900}$.