Properties of Logarithms

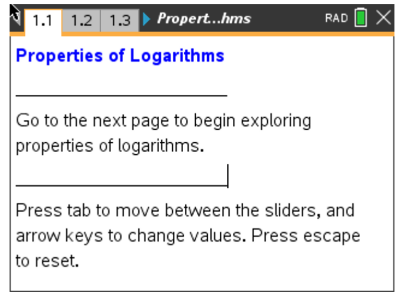
Student Activity

Name ________________

Class ________________

Open the TI-Nspire document *Properties_of_Logarithms.tns.*

This activity explores the product property, the quotient property, and the power property of logarithms both algebraically and graphically.



Move to page 1.2.

For this activity, the expression used is $\log_2(x)$. The investigations also work for any base > 0 and base ≠ 1.

1. As you drag the sliders for *m* and *n,* note what happens as these values are substituted into the four expressions.
   a. Find which expressions, if any, appear to be equivalent independent of the values of *m* and *n.*

   b. Set *m* = 8 and *n* = 4. Substitute these values into the logarithmic expressions you found to be equivalent in part 1a, and simplify these expressions to show they are indeed equivalent.

   c. Use the expressions you found in parts 1a and 1b to write a general logarithmic property for $\log_a mn$, where *a* is a real number, $a > 0$ and $a \neq 1$.

   d. Explain how the operations in the logarithmic property in part 1c relate to the operations in the exponential property $a^m a^n = a^{m+n}$.

Move to page 1.5

Now let's look at this same idea but graphically. Suppose you wanted to simplify the logarithm of a product, like $\log 6a$. Think about how you might go about doing this. Let's start by defining a new variable $\boldsymbol{b = 6a}$.

Step a: At the top of column 1, name this list $\boldsymbol{a}$. Enter at least 10 values for $\boldsymbol{a}$, that are in the domain of the logarithmic function.

Step b: At the top of column 2, name this list $\boldsymbol{b}$. Move down to the second row and enter a formula that will calculate $\boldsymbol{b = 6a,}$ from the values in column 1.

Step c: Move to **page 1.6** and click on the bottom to add variable $\boldsymbol{a}$ and click on the left to add variable $\boldsymbol{b}$.

2. Describe the shape of the graph. Discuss with a classmate if it is what you expected. Share your results with the class.

Step d: Move back to **page 1.5**. Now we will define two new variables, x and y. Let $\boldsymbol{x = \log a}$ and $\boldsymbol{y = \log b}$. At the top of the third column, name it $\boldsymbol{x}$. Move down to the second row and enter a formula that calculates $\boldsymbol{x}$ from the values in column 1. At the top of the fourth column, name it $\boldsymbol{y}$. Move down to the second row and enter a formula that calculates $\boldsymbol{y}$ from the values in column 2.

Step e: Move to **page 1.7** and click on the bottom to add variable $\boldsymbol{x}$ and click on the left to add variable $\boldsymbol{y}$.

The data appear linear. Find the equation of a line through these points by pressing **menu**, **4 Analyze**, **6 Regression**, **1 Linear (mx + b)**.

3. Write down the equation of the line through these points.

4. Find the y-intercept of the line.

You should have found that the equation of the line was $y = x + 0.778151$. Think about where this $y - intercept$ comes from. (Here's a hint: Try raising 10 to the 0.778151 power.)

5. Using logs, find what 0.778151 is.

6. Since $10^{0.778151} \approx$ ___________, $\log(6) \approx$ ____________.

You have found that $y = \log 6 + x$. Think about what this means. Substitute to rewrite this as an equation in terms of $\boldsymbol{a}$. The explanation for each step is given to the right.

$y = \log 6 + x$	Equation of the line
	$x = \log a$ and $y = \log b$
	$b = 6a$

Product Property of Logarithms

For $a > 0$ and $b > 0$, $\log ab = \log a + \log b$.

Examples $\log xy$ is written in *expanded form* as $\log x + \log y$

$\log 7 + \log z$ is written as a single logarithm as $\log 7z$

Move to page 1.3

7. As you drag the sliders for *m* and *n,* note what happens as these values are substituted into the four expressions.
   a. Find which expressions, if any, appear to be equivalent independent of the values of *m* and *n.*

   b. Set *m* = 8 and *n* = 4. Substitute these values into the logarithmic expressions you found to be equivalent in part 7a, and simplify these expressions to show they are indeed equivalent.

   c. Use the expressions you found in parts 7a and 7b to write a general logarithmic property for $\log_a\left(\frac{m}{n}\right)$ where *a* is a real number, $a > 0$ and $a \neq 1$.

   d. Explain how the operations in the logarithmic property in part 7c relate to the operations in the exponential property $\frac{a^m}{a^n} = a^{m-n}$.

Name _______________

Class _______________

Move to page 2.1

Again, let's look at this same idea but graphically. Suppose you wanted to simplify the logarithm of a quotient, like $\log\frac{8}{a}$. Think about how you might go about doing this. Let's start by defining a new variable $\boldsymbol{b} = \frac{\mathbf{8}}{\boldsymbol{a}}$.

Step a: At the top of column 1, name this list $\boldsymbol{a}$. Enter at least 10 values for $\boldsymbol{a}$, that are in the domain of the logarithmic function.

Step b: At the top of column 2, name this list $\boldsymbol{b}$. Move down to the second row and enter a formula that will calculate $\boldsymbol{b} = \frac{\mathbf{8}}{\boldsymbol{a}}$, from the values in column 1.

Step c: Move to **page 2.2** and click on the bottom to add variable $\boldsymbol{a}$ and click on the left to add variable $\boldsymbol{b}$.

8. Describe the shape of the graph. Discuss with a classmate if it is what you expected. Share your results with the class.

Step d: Move back to page 2.1. Now we will define two new variables, x and y. Let $\boldsymbol{x} = \mathbf{log}\,\boldsymbol{a}$ and $\boldsymbol{y} = \mathbf{log}\,\boldsymbol{b}$. At the top of the third column, name it $\boldsymbol{x}$. Move down to the second row and enter a formula that calculates $\boldsymbol{x}$ from the values in column 1. At the top of the fourth column, name it $\boldsymbol{y}$. Move down to the second row and enter a formula that calculates $\boldsymbol{y}$ from the values in column 2.

Step e: Move to **page 2.3** and click on the bottom to add variable $\boldsymbol{x}$ and click on the left to add variable $\boldsymbol{y}$.

The data appear linear. Find the equation of a line through these points by pressing **menu**, **4 Analyze**, **6 Regression**, **1 Linear (mx + b)**.

9. Write down the equation of the line through these points.

10. Find the y-intercept of the line.

You should have found that the equation of the line was $y = 0.90309 - x$. Think about where this $y - intercept$ comes from.

Name ______________
Class ______________

11. Using logs, find what 0.90309 is.
12. Since $10^{0.90309} \approx$ ___________, $\log(8) \approx$ ____________.

You have found that $y = \log 8 - x$. Think about what this means. Substitute to rewrite this as an equation in terms of $\boldsymbol{a}$. The explanation for each step is given to the right.

$y = \log 8 - x$	Equation of the line
	$x = \log a$ and $y = \log b$
	$b = \dfrac{8}{a}$

Quotient Property of Logarithms

For $a > 0$ and $b > 0$, $\log ab = \log a + \log b$.

Examples $\log \frac{x}{y}$ is written in *expanded form* as $\log x - \log y$

$\log 7 - \log z$ is written as a single logarithm as $\log \frac{7}{z}$

Move to page 1.4

13. As you drag the sliders for *m* and *n,* note what happens as these values are substituted into the three expressions.
    a. Find which expressions, if any, appear to be equivalent independent of the values of *m* and *n.*

    b. Set *m* = 4 and *n* = 3. Substitute these values into the logarithmic expressions you found in part 13a, and simplify these expressions to show they are equivalent.

    c. Use the expressions you found in parts 13a and 13b to write a general logarithmic property for $\log_a (m)^n$ where *a* is a real number, $a > 0$ and $a \neq 1$

    d. Explain how the operations in the logarithmic property in part 3c relate to the operations in the exponential property $\left(a^m\right)^n = a^{mn}$.

e. Use the logarithmic property you proved in part 13c to show that $\log_a a = 1$ for all values of *a* where $a > 0$ and $a \neq 1$.

f. Use the logarithmic property you proved in part 13c to show that $\log_a 1 = 0$ for all values of *a* where $a > 0$ and $a \neq 1$.

Move to page 3.1

One final time, let's look at this same idea but graphically. Suppose you wanted to simplify the logarithm of a power, like $\log a^2$. Think about how you might go about doing this. Let's start by defining a new variable $\boldsymbol{b = a^2}$.

Step a: At the top of column 1, name this list $\boldsymbol{a}$. Enter at least 10 values for $\boldsymbol{a}$, that are in the domain of the logarithmic function.

Step b: At the top of column 2, name this list $\boldsymbol{b}$. Move down to the second row and enter a formula that will calculate $\boldsymbol{b = a^2}$, from the values in column 1.

Step c: Move to **page 3.2** and click on the bottom to add variable $\boldsymbol{a}$ and click on the left to add variable $\boldsymbol{b}$.

14. Describe the shape of the graph. Discuss with a classmate if it is what you expected. Share your results with the class.

Step d: Move back to page 3.1. Now we will define two new variables, x and y. Let $\boldsymbol{x = \log a}$ and $\boldsymbol{y = \log b}$. At the top of the third column, name it $\boldsymbol{x}$. Move down to the second row and enter a formula that calculates $\boldsymbol{x}$ from the values in column 1. At the top of the fourth column, name it $\boldsymbol{y}$. Move down to the second row and enter a formula that calculates $\boldsymbol{y}$ from the values in column 2.

Step e: Move to **page 3.3** and click on the bottom to add variable $\boldsymbol{x}$ and click on the left to add variable $\boldsymbol{y}$.

The data appear linear. Find the equation of a line through these points by pressing **menu**, **4 Analyze**, **6 Regression**, **1 Linear (mx + b)**.

15. Write down the equation of the line through these points.

16. Find the y-intercept of the line.

You should have found that the equation of the line was $y = 2x$. Think about what this means.

You have found that $y = \log 6 + x$. Think about what this means. Substitute to rewrite this as an equation in terms of $\boldsymbol{a}$. The explanation for each step is given to the right.

$y = 2x$	Equation of the line
$\log b = 2\log a$	$x = \log a$ and $y = \log b$
$\log a^2 = 2\log a$	$b = a^2$

Power Property of Logarithms
For $a > 0$, $\log a^b = b\log a$.

Examples $\log x^3$ can be written as $3\log x$
$8\log x$ can be written as $\log x^8$

<u>Further IB Math Extension</u>

Using the properties discussed in this activity, find the solution of:

$$\log_3 x - 2\log_3 2 = 3 - \log_3 2$$