
Using the Document

This tns file is used to produce the graphs of various Taylor polynomials, $y = T_n(x)$. These graphs are used to study how the accuracy of a Taylor polynomial is associated with the degree of the Taylor polynomial. The accuracy of each Taylor polynomial is visualized and can be related to symmetry, arc length, and any points of discontinuity.

Five common functions are considered and some specific questions about associated Taylor polynomials are included.

Problems

1. For $a = 0$, sketch and describe the graph of $y = T_1(x)$.
2. Use the graph of $y = T_1(x)$ and the Trace All feature to describe the accuracy of the Taylor polynomial approximation as x moves farther away from $a = 0$.
3. Set $n = 2$. Sketch and describe the graph of $y = T_2(x)$.
4. Set $n = 3$. Sketch and describe the graph of $y = T_3(x)$.
5. Consider the graph of other Taylor polynomials for $n \geq 4$. Describe the accuracy of the Taylor polynomial approximation as n increases.

Problems

Change the values of n and a as necessary and use **Page 2.3**, the Lists and Spreadsheet page, to answer the following questions.

1. For a fixed value of n , describe the accuracy of the Taylor polynomial approximation as the values of x are farther away from a .
2. For fixed values of a and x , describe the accuracy of the Taylor polynomial approximation as n increases.
3. Set $a = 0$. For $n = 1, 2, 3, 4, 5$, find an interval in which the Taylor polynomial is a good approximation for f .

Note: Page 2.4 is used for background calculations. The equations and entries on this Lists and Spreadsheet page should be left unchanged to ensure the accuracy of the results presented on other pages of this problem.

Problems

Change the values of n and a as necessary and use **Page 3.2** to answer the following questions.

1. For $a = 2$, describe the accuracy of the Taylor polynomial approximation as n increases.
2. Describe the behavior of each Taylor polynomial as $x \rightarrow +\infty$ and as $x \rightarrow -\infty$. Explain how the graph of the Taylor polynomial changes, as $x \rightarrow +\infty$, as n increases by 1, for example, from $n = 6$ to $n = 7$. Explain why this property of the Taylor polynomials alternates as n increases.
3. Set $a = 0.3$. Consider the graph of $y = T_n(x)$ for various values of n . Explain why the Taylor polynomial appears to be a very good approximation to the left of $a = 0.3$ but diverges rapidly to the right of $a = 0.3$.

Problems

Change the values of n and a as necessary and use **Pages 5.2 and 5.3** to answer the following questions.

1. Set $a = 0$ and $n = 1$. Describe the graph of the Taylor polynomial. Find the Taylor polynomial and explain why the slope of this linear approximation is 0.
2. Set $a = 0$. Consider the graph of the Taylor polynomial $y = T_n(x)$ as n increases. Explain why the graph of the Taylor polynomials for $n = 0$ and $n = 1$ are identical, and for $n = 2$ and for $n = 3$, etc.

Problems

Change the values of n and a as necessary and use **Pages 6.2 and 6.3** to answer the following questions.

1. Set $a = 0$. Consider the graph of the Taylor polynomial $y = T_n(x)$ for various values of n . Explain why there is no graph of the Taylor polynomial to the right of $x = 1$.
2. Set $a = 0$ and $n = 7$. Explain the accuracy of the Taylor polynomial. Explain the accuracy of the Taylor polynomial $T_7(x)$. Why does the Taylor polynomial appear to be a much better approximation to the right of $x = 0$ than to the left?
3. Explain how to obtain the graph of a Taylor polynomial that can be used to approximate the portion of the graph of $y = \frac{1}{1-x}$ to the right of $x = 1$.

Suggested Extensions

1. Explain how the accuracy of a Taylor polynomial is related to the degree of the Taylor polynomial and the value of a .
2. Describe the interval about $x = a$ on which a Taylor polynomial is fairly accurate.
3. Suppose the function f has a discontinuity at $x = b$. Explain how this value affects a Taylor polynomial for f .
4. What is the relationship between the i th derivative of the function f and the i th derivative of the corresponding Taylor polynomial T_n ?
5. Consider exploring the Taylor polynomials associated with the following functions.
 - (a) $f(x) = \tan^{-1} x$
 - (b) $f(x) = e^{-x^2}$
 - (c) $f(x) = e^x \sin x$
 - (d) $f(x) = \frac{1}{1+x^2}$
 - (e) $f(x) = \tan x$