Math Objectives

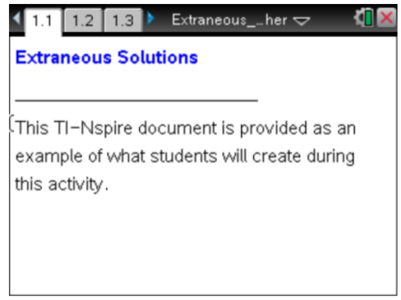
- Students will determine the solution of radical equations both algebraically and geometrically.
- Students will know why solutions to radical equations should be checked by algebraic solution for extraneous solutions.
- Students will establish the relationship between real and nonreal number solutions for radical equations.
- Students will be able to explain why squaring both sides of an equation can lead to extraneous solutions.

Vocabulary

- radical equations
- real numbers
- extraneous solutions

About the Lesson

- This lesson allows students to discover solutions of radical equations and investigate extraneous solutions.
- As a result, students will:
    - Solve radical equations using handheld graphing technology
    - Solve radical equations using algebra and test the validity of the solution
    - Justify the solution(s) of a radical equation



TI-Nspire™ Technology Skills:

- Download a TI-Nspire™ document
- Open a document
- Move between pages
- Graph functions
- Evaluate functions using handheld calculation tools

Tech Tips:

- Make sure the font size on your TI-Nspire™ handheld is set to Medium.
- Once a function has been graphed, the entry line can be graphed by pressing ctrl G. The entry line can also be expanded or collapsed by clicking the chevron.

Lesson Materials:
Student Activity
Extraneous_Solutions_Student.pdf
Extraneous_Solutions_Student.doc

TI-Nspire™ document
Extraneous_Solutions_Teacher.tns

Visit www.mathnspired.com for lesson updates and tech tip videos. (optional)

Discussion Points and Possible Answers

Teacher Tip: The directions for this activity are step-by-step. If you wish to move more quickly through the activity, you may omit many of the step-by-step directions. A teacher TI-Nspire™ document is also supplied.

Teacher Tip: Question 1 is meant to be a lead-in to the activity. Students are encouraged to do this question without the use of the technology.

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1. Solve the radical equation $\sqrt{2-x} = x$ algebraically and record your solutions.

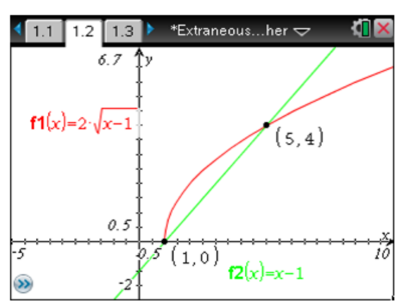
   a. How many solutions did you find for the equation? Check algebraically to determine whether these solutions are correct?

   Answer: Students should find two solutions using an algebraic approach (squaring both sides and solving). If they substitute both answers into the original equation, they should find that $x =$ -2 is not a solution.

   b. Visualize each side of the equation as separate functions. What does the graph of $f(x) = \sqrt{2-x}$ look like? What does the graph of $f(x) = x$ look like? In how many points might these two graphs intersect? How does this compare to your answer in part 1a?

   Sample answer: The first function is a square-root function that starts at the point (2, 0) and goes to the left. The second function is a straight line at a 45° angle from the *x*-axis. The functions intersect at one point. This confirms the single solution found in question 1a.

Press [on] **> New Document > Add Graphs**.



2. In the entry line, enter the function $f1(x) = 2\sqrt{x-1}$ and press [enter]. Then press [ctrl] [G] to enter another function: $f2(x) = x-1$.

   a. How many points of intersection are there for these two functions?

   Answer: There are two points of intersection.

b. What do you know about the points of intersection for these two functions?

<u>Sample answer:</u> Students should recognize that the points of intersection of the graphs are solutions to the equation made by setting the two functions equal to each other.

3. To find the points of intersection, select **Menu > Geometry > Points & Lines > Intersection Point(s)**. Move the cursor on the handheld to both graphs and press [enter] on each graph. Where are the points of intersection?

<u>Answer:</u> The points of intersection are (1, 0) and (5, 4).

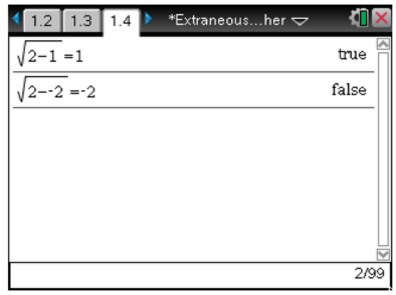
> **Teacher Tip:** TI-Nspire™ CAS may be used to solve equations as well as to check for truth values in solutions. For this activity, only the TI-Nspire™ was used in the exploration of extraneous solutions.

4. An algebraic solution is necessary to prove that the intersection points are actually solutions to the equation. To solve the equation algebraically and check your solutions, add a new *Calculator* page to the document by selecting [ctrl] [doc▾]. Check whether the points of intersection are true solutions to the equation by substituting each *x*-value into the equation $2\sqrt{x-1} = x-1$. and enter the equation into the handheld. For example, check whether (5, 4) is a point of intersection. Enter a similar equation to check the other point of intersection.

Did the truth value (either true or false) confirm your algebraic solution and the solution obtained by using the intersection tool? Explain.

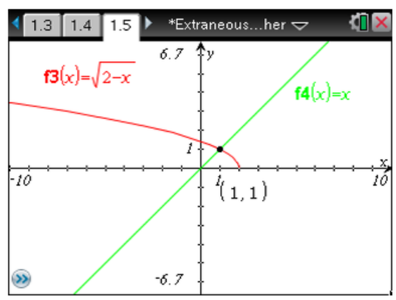
<u>Answer:</u> The algebraic solutions should match the solutions obtained from the graphical representation of the equation. Students should recognize there are two solutions since the functions intersect twice.

5. Let's revisit the equation from question 1, $\sqrt{2-x}=x$. Check your solutions as you did in question 4 by adding a new *Calculator* page to the document and entering the equation. Remember that *true* implies the solution is correct, while *false* implies the solution is incorrect. Did the truth value confirm your solution algebraically and the solution obtained graphically? Explain.



<u>Answer:</u> Solutions to the algebraic solution should return a true statement for $x=1$ but a false statement for $x=-2$. The reason for this is that squaring both sides introduces a root that falls below the *x*-axis and therefore cannot be in the range of $f(x)=\sqrt{2-x}$.

6. To find the graphical solution of this equation, graph each side of the equation as a separate function. Add a new *Graphs* page to the document. Then enter $f3(x)=\sqrt{2-x}$. Press ctrl G to bring back the entry line to enter $f4(x)=x$. To find the point(s) of intersection, press **Menu > Geometry > Points & Lines > Intersection Point(s).** Select each graph and all intersections points will show. Record the point(s) of intersection.



a. What do the point(s) of intersection of the two functions represent and where do they occur? Explain.

<u>Answer:</u> The point of intersection (1, 1) represents the only point that the two functions share. The *x*-coordinate at the point of intersection represents the solution to the equation when the functions are set equal to each other.

b. How does the solution for this equation differ from the equation in question 2? Explain your reasoning.

<u>Answer:</u> Students should recognize that there is only one solution to the equation instead of two solutions as in question 2.

c. Explain why there are not two solutions, as in question 2.

<u>Answer:</u> Additional solutions are introduced by squaring both sides of the equation.

d. Describe why and when the solutions of a radical equation need to be checked.

Answer: When raising each side of the equation to a power, solutions that are not true for the original equation may be introduced. When solving equations where you raise both sides to a power, you need to check your solutions.

> **Teacher Tip:** In general, you want the students to express the idea that raising each side of the equation to a power might introduce solutions that are not true for the original equation. When solving equations where you raise both sides to a power, you should check your solutions. As an example, you can explain to them that if $x – 3 = 4$, then $x = 7$. However, if you square both sides of the equation $x – 3 = 4$, you get $(x – 3)^2 = 16$, which gives results $x = 7$ and $x = –1$. Students should begin to see that squaring both sides introduces an additional root, which may be an extraneous solution.

> **Teacher Tip:** You might want to discuss the following questions with students: Does adding, subtracting, multiplying, or dividing (by a nonzero value) introduce extraneous roots? Does a vertical translation or a vertical dilation change the intersection points? Does either one change the number of intersection points or the location?

7. It is important to check the solutions to equations containing a radical because some may be extraneous solutions.

a. What is an extraneous solution of an equation?

Answer: Extraneous solutions of an equation are solutions that are introduced when a radical expression with an even index, such as 2, is raised to its power to solve an equation.

b. Why do extraneous solutions sometimes occur in the process of solving rational or radical equations? Explain your reasoning.

Answer: Squaring both sides of a square-root radical equation introduces two solutions (the positive and negative) that might be the solution to the equation.

> **Teacher Tip:** As an extension, it might be interesting to discuss the possibility of the number of solutions to radical equations based on the size of the index and incorporate complex roots as possible solutions.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to solve radical equations using technology
- How to justify the solution to radical equations
- How extraneous roots are produced from radical equations