1.

(a) Using the definition of a derivative as $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, show that the derivative of $\frac{2}{3x-5}$ is $\frac{-6}{(3x-5)^2}$. (4 marks)

(b) Using the same function, $\frac{2}{3x-5}$, find $\lim_{h\to 0}\frac{f(x+h)-f(x)}{2h}$. Explain why this limit may be used as a better approximation that the limit used in part (a). (4 marks)

Mark scheme:

(a) $\lim_{h\to 0}\frac{\frac{2}{3(x+h)-5}-\frac{2}{3x-5}}{h}$ (M1)(A1)

$$\lim_{h\to 0}\frac{\frac{2}{3x+3h-5}*\frac{3x-5}{3x-5}-\frac{2}{3x-5}*\frac{3x+3h-5}{3x+3h-5}}{h}$$ (A1)

$$\lim_{h\to 0}\frac{\frac{6x-10-6x-6h+10}{(3x+3h-5)(3x-5)}}{h}$$

$$\lim_{h\to 0}\frac{-6h}{(3x+3h-5)(3x-5)}*\frac{1}{h}$$ (A1)

$$=\frac{-6}{(3x-5)^2}$$ (AG)

(b) $\lim_{h\to 0}\frac{\frac{2}{3(x+h)-5}-\frac{2}{3(x-h)-5}}{2h}$ (A1)

$$\lim_{h\to 0}\frac{\frac{2}{3x+3h-5}*\frac{3x-3h-5}{3x-3h-5}-\frac{2}{3x-5}*\frac{3x+3h-5}{3x+3h-5}}{2h}$$ (A1)

$$\lim_{h\to 0}\frac{\frac{6x-6h-10-6x-6h+10}{(3x+3h-5)(3x-3h-5)}}{2h}$$

$$\lim_{h \to 0} \frac{-12h}{(3x+3h-5)(3x-3h-5)} * \frac{1}{2h}$$

$$\lim_{h \to 0} \frac{-6}{(3x+3h-5)(3x-3h-5)}$$

$$= \frac{-6}{(3x-5)^2}$$ (A1)

When finding the slope of a tangent at a point on a curve, this symmetric difference quotient evaluates to the right AND to the left of that point, whereas the difference quotient just evaluates to the left OR the right of that point. (R1)